W IX

Rozważamy przykład tzw. chiralnego potencjału, czyli potencjału nukleon-nukleon wyprowadzonego w ramach chiralnej efektywnej teorii pola. Evgeny Epelbaum i współpracownicy z Bochum i Bonn wprowadzili w rzędzie wiodącym (LO) wyrażenie

$$\langle \vec{p}\,' | \hat{V} | \vec{p} \rangle = \left\{ -\frac{g_A^2}{4F_\pi^2} \frac{\vec{q}\cdot\vec{\sigma}_1 \; \vec{q}\cdot\vec{\sigma}_2}{\vec{q}^{\,2}+m_\pi^2} \vec{\tau}_1\cdot\vec{\tau}_2 + C_S \mathbb{1}^{spin} \otimes \mathbb{1}^{izospin} + C_T \vec{\sigma}_1\cdot\vec{\sigma}_2 \otimes \mathbb{1}^{izospin} \right\} \cdot F_\Lambda(\vec{p}\,', \vec{p})$$

$\vec{q} = \vec{p}\,' - \vec{p}$

$g_A \simeq 1.29$ – stała sprzężenia aksjalno-wektorowego

$F_\pi \simeq 92.4$ MeV – stała rozpadu pionu

C_S, C_T – liczby

$\mathbb{1}^{spin}$ ($\mathbb{1}^{izospin}$) – operator identyczności w przestrzeni spinowej (izospinowej)

$F_\Lambda(\vec{p}\,', \vec{p})$ – funkcja regulująca wprowadzona w celu "obcięcia" dużych wartości pędów. W pierwszych wersjach $F_\Lambda(\vec{p}\,', \vec{p}) = f_\Lambda(|\vec{p}\,'|)\, f_\Lambda(|\vec{p}|)$

$$\langle p \alpha_2 | \hat{V} | p' \alpha_2' \rangle = \int d^3 \vec{p_1} \int d^3 \vec{p_2}$$

$$\langle p \alpha_2 | \vec{p_1} \rangle \langle \vec{p_1} | \hat{V} | \vec{p_1}' \rangle \langle \vec{p_1}' | p' \alpha_2' \rangle = \ldots =$$

$$= \int d\hat{p} \int d\hat{p}' \sum_{m_l} \sum_{m_l'} c(l s j; m_l, m_j - m_l, m_j)$$

$$c(l' s' j'; m_l', m_j' - m_l', m_j')$$

$$Y^*_{l m_l}(\hat{p}) \, Y_{l' m_l'}(\hat{p}')$$

$$\langle t m_t | \langle s\, m_j - m_l | \langle \vec{p} | \hat{V} | \vec{p}' \rangle | s'\, m_j' - m_l' \rangle | t' m_t' \rangle$$

Można wykorzystać bezpośrednio ten wzór w tzw. automatycznej metodzie rozkładu na fale parcjalne: Eur. Phys. J. A 43, 241 (2010)

Metoda ta sprowadza się do policzenia elementów macierzowych

$$\langle t m_t | \langle s m_s | \langle \vec{p} | \hat{V} | \vec{p}' \rangle | s' m_s' \rangle | t' m_t' \rangle$$

przy pomocy programu Mathematica i numerycznym liczeniu wielu poczwórnych całek.

Zamiast tego chcemy uzyskać analityczne wyrażenie dla $\langle p\alpha_2 | V | p'\alpha_2' \rangle$. W tym celu wykonamy bardzo wiele przesprzężeń, wykorzystamy właściwości współczynników Clebscha-Gordana, rozwinięcie funkcji skalarnej na wielomiany Legendre, ortogonalności harmonik sferycznych, ...

Ogólna notacja:

$$\{A_{j_1} B_{j_2}\}^{jm} \equiv \sum_{m_1} c(j_1 j_2 j; m_1, m-m_1, m) A_{j_1 m_1} B_{j_2 m-m_1}$$

Składowe sferyczne wektora

$$(\vec{a})_\tau = \sqrt{\frac{4\pi}{3}} |\vec{a}| Y_{1\tau}(\hat{a})$$

$$\vec{a} \cdot \vec{b} = -\sqrt{3} \{a_1 b_1\}^{00}$$

$$(\vec{a} \times \vec{b})_\tau = -i\sqrt{2} \{a_1 b_1\}^{1\tau} \equiv$$

$$\equiv -i\sqrt{2} \sum_m c(111; m, \tau-m, \tau) a_m b_{\tau-m}$$

notacja $\hat{x} \equiv 2x+1$ (dla liczb całkowitych i połówkowych)

$$\vec{\sigma_1}\cdot\vec{q} = -\sqrt{3}\,\{\sigma_1 q_1\}^{00}$$

$$\vec{\sigma_2}\cdot\vec{q} = -\sqrt{3}\,\{\sigma_2 q\}^{00}$$

$$(\vec{\sigma_1}\cdot\vec{q})(\vec{\sigma_2}\cdot\vec{q}) = 3\,\{\sigma_1 q_1\}^{00}\{\sigma_2 q_1\}^{00}$$

$$= 3\left\{\{\underset{j_1}{\sigma_1}\underset{j_2}{q_1}\}^{\overset{J_{12}}{0}}\{\underset{j_3}{\sigma_2}\underset{j_4}{q_1}\}^{\overset{J_{34}}{0}}\right\}^{\overset{JM}{00}} =$$

$$= 3\sum_{a,b}\sqrt{\hat{0}\hat{0}\hat{a}\hat{b}}\begin{Bmatrix}1&1&0\\1&1&0\\a&b&0\end{Bmatrix}$$

$$\left\{\{\sigma_1\sigma_2\}^a\{q_1 q_1\}^b\right\}^{00}$$

$$= 3\sum_a \hat{a}\begin{Bmatrix}1&1&0\\1&1&0\\a&a&0\end{Bmatrix}\left\{\{\sigma_1\sigma_2\}^a\{q_1 q_1\}^a\right\}^{00}$$

$$\begin{Bmatrix}1&1&0\\1&1&0\\a&a&0\end{Bmatrix} = \frac{(-1)^{1+1+a}}{\sqrt{\hat{a}}}\begin{Bmatrix}1&1&0\\1&1&a\end{Bmatrix} =$$

$$= \frac{(-1)^a}{\sqrt{\hat{a}}}\begin{Bmatrix}1&1&a\\1&1&0\end{Bmatrix} = \frac{(-1)^a}{\sqrt{\hat{a}}}(-1)^{1+1+a}\frac{1}{\sqrt{\hat{1}\hat{1}}}$$

$$= \frac{1}{3\sqrt{\hat{a}}} \qquad (4)$$

$$(\vec{\sigma_1}\cdot\vec{q})(\vec{\sigma_2}\cdot\vec{q}) = \sum_a\sqrt{\hat{a}}\left\{\{\sigma_1\sigma_2\}^a\{q_1 q_1\}^a\right\}^{00}$$

$$q_{1\tau} = \sqrt{\frac{4\pi}{3}}\,|\vec{q}|\,Y_{1\tau}(\hat{q})$$

$$(\vec{\sigma}_1\cdot\vec{q})(\vec{\sigma}_2\cdot\vec{q}) = \frac{4\pi}{3}|\vec{q}|^2$$

$$\sum_a \sqrt{\hat{a}}\left\{\{\sigma_1\sigma_2\}^a\{Y_1(\hat{q})Y_1(\hat{q})\}^a\right\}^{00}$$

$$\{Y_1(\hat{q})Y_1(\hat{q})\}^a \equiv \mathcal{Y}_{11}^{a}(\hat{q}\hat{q}) =$$

$$= \sqrt{\frac{9}{4\pi\hat{a}}}\,(11a;000)\,Y_a(\hat{q})$$

$$\mathcal{Y}_{l_1l_2}^{lm}(\hat{a}\hat{a}) = \sqrt{\frac{\hat{l}_1\hat{l}_2}{\hat{l}\,4\pi}}\,(l_1l_2l;000)\,Y_{lm}(\hat{a}) \quad (5)$$

$$(\vec{\sigma}_1\cdot\vec{q})(\vec{\sigma}_2\cdot\vec{q}) = \sqrt{4\pi}\,|\vec{q}|^2\sum_a (11a;000)$$

$$\left\{\{\sigma_1\sigma_2\}^a Y_a(\hat{q})\right\}^{00}$$

$$\vec{q} = \vec{p}\,' - \vec{p} = \vec{p}\,' + (-\vec{p})$$

$$Y_a(\hat{q}) = \sum_{a_1+a_2=a} \frac{(p')^{a_1}p^{a_2}}{|\vec{q}|^a}\sqrt{\frac{4\pi(2a+1)!}{(2a_1+1)!\,(2a_2+1)!}}$$

$$\mathcal{Y}_{a_1a_2}^{a}(\hat{p}'\hat{p})(-1)^{a_2} \qquad (6)$$

$$\frac{(\vec{\sigma}_1\cdot\vec{q})(\vec{\sigma}_2\cdot\vec{q})}{m_\pi^2+\vec{q}^{\,2}} = (\sqrt{4\pi})^2 \sum_a (11a;000)$$

$$\sqrt{(2a+1)!}\sum_{a_1+a_2=a}(p')^{a_1}p^{a_2}\frac{(-1)^{a_2}}{\sqrt{(2a_1+1)!(2a_2+1)!}}$$

$$\left\{\{\sigma_1\sigma_2\}^a\,\mathcal{Y}^a_{a_1a_2}(\hat{p}'\hat{p})\right\}^0 \underbrace{\frac{|\vec{q}|^{2-a}}{m_\pi^2+\vec{q}^{\,2}}}_{h(p',p,\underbrace{\hat{p}'\cdot\hat{p}}_{x})}$$

$h(p',p,x)$ rozwijamy na wielomiany Legendre

$$h(p',p,\hat{p}'\cdot\hat{p}) = 2\pi\sum_{k=0}^{\infty}\sqrt{\hat{k}}\,(-1)^k g_{ka}\cdot\mathcal{Y}^{00}_{kk}(\hat{p}'\hat{p}), \qquad (7)$$

$$g_{ka} = \int_{-1}^{1}dx\, P_k(x)\, h(p',p,x)$$

$$\frac{(\vec{\sigma}_1\cdot\vec{q})(\vec{\sigma}_2\cdot\vec{q})}{m_\pi^2+\vec{q}^{\,2}} = 8\pi^2\sum_a (11a;000)\sqrt{(2a+1)!}$$

$$\sum_{a_1+a_2=a}(p')^{a_1}p^{a_2}(-1)^{a_2}\frac{1}{\sqrt{(2a_1+1)!(2a_2+1)!}}$$

$$\sum_k\sqrt{\hat{k}}\,(-1)^k g_{ka}$$

$$\left\{\left\{\{\sigma_1\sigma_2\}^{a}_{j1}\,\mathcal{Y}^{a\,j2}_{a_1a_2}(\hat{p}'\hat{p})\right\}^{j12}_{0}\,\mathcal{Y}^{0\,j3}_{kk}(\hat{p}'\hat{p})\right\}^{J}_{00}$$

$$\frac{(\vec{\sigma}_1\cdot\vec{q})(\vec{\sigma}_2\cdot\vec{q})}{m_\pi^2+\vec{q}^{\,2}} = 8\pi^2 \sum_a (11a;000)\sqrt{(2a+1)!}$$

$$\sum_{a_1+a_2=a} (p')^{a_1}\, p^{a_2} \frac{(-1)^{a_2}}{\sqrt{(2a_1+1)!\,(2a_2+1)!}}$$

$$\sum_k \sqrt{\hat{k}}\,(-1)^k g_{ka}$$

$$\sum_b \sqrt{\hat{b}}\;(-1)^{a+a+0+0} \begin{Bmatrix} a & a & 0 \\ 0 & 0 & b \end{Bmatrix}$$

$$\left\{ \{\sigma_1\sigma_2\}^a \left\{ \mathcal{Y}^a_{a_1a_2}(\hat{p}'\hat{p})\, \mathcal{Y}^0_{kk}(\hat{p}'\hat{p}) \right\}^b \right\}^{00}$$

$$\begin{Bmatrix} a & a & 0 \\ 0 & 0 & b \end{Bmatrix} = (-1)^{a+a+0} \frac{1}{\sqrt{\hat{b}}}\,\delta_{ab} \qquad (8)$$

$$\frac{(\vec{\sigma}_1\cdot\vec{q})(\vec{\sigma}_2\cdot\vec{q})}{m_\pi^2+\vec{q}^{\,2}} = 8\pi^2 \sum_a (11a;000)\sqrt{(2a+1)!}$$

$$\sum_{a_1+a_2=a} (p')^{a_1}\, p^{a_2} \frac{(-1)^{a_2}}{\sqrt{(2a_1+1)!\,(2a_2+1)!}}$$

$$\sum_k \sqrt{\hat{k}}\,(-1)^k g_{ka}$$

$$\left\{ \{\sigma_1\sigma_2\}^a \left\{ \underset{j1\,j2}{\overset{j12}{\mathcal{Y}^a_{a_1,a_2}}}(\hat{p}'\hat{p})\, \underset{j3\,j4}{\overset{j34}{\mathcal{Y}^0_{kk}}}(\hat{p}'\hat{p}) \right\}^{\overset{J}{a}} \right\}^{00}$$

$$\mathcal{Y}^{a}_{a_1 a_2}(\hat{p}'\hat{p})\,\mathcal{Y}^{0}_{kk}(\hat{p}'\hat{p}) = \dots$$

Glöckle Eq. (A.16) pp. 163–164

$$= \frac{1}{4\pi}\sqrt{\hat{k}\,\hat{a}_1\hat{a}_2}\,(-1)^{a_1+a_2+a}$$

$$\sum_{f_1,f_2} \begin{Bmatrix} f_2 & f_1 & a \\ a_1 & a_2 & k \end{Bmatrix} (k a_1 f_1;000)(k a_2 f_2;000)\,\mathcal{Y}^{a}_{f_1 f_2}(\hat{p}'\hat{p})$$

$$\frac{(\vec{\sigma}_1\cdot\vec{q})(\vec{\sigma}_2\cdot\vec{q})}{m_\pi^2+\vec{q}^{\,2}} = 2\pi \sum_{a=0,2} (11a;000)\sqrt{(2a+1)!}$$

$$\sum_{a_1+a_2=a} (p')^{a_1}\, p^{a_2}\,(-1)^{a_2}\,\frac{\sqrt{\hat{a}_1\hat{a}_2}}{\sqrt{(2a_1+1)!\,(2a_2+1)!}}$$

$$\sum_{k} \sqrt{\hat{k}}\,(-1)^k\, g_{ka}\sqrt{\hat{k}}$$

$$\sum_{f_1 f_2} \begin{Bmatrix} f_2 & f_1 & a \\ a_1 & a_2 & k \end{Bmatrix} (k a_1 f_1;000)(k a_2 f_2;000)$$

$$\left\{ \{\sigma_1\sigma_2\}^{a}\,\mathcal{Y}^{a}_{f_1 f_2}(\hat{p}'\hat{p}) \right\}^{00}$$

$(j_1 j_2 j_3;000) = 0$, if $j_1+j_2+j_3$ is odd

(9)

$$\left\{ \{\sigma_1\sigma_2\}^a \mathcal{Y}^a_{f_1f_2}(\hat{p}'\hat{p}) \right\}^{00} =$$

$$= \sum_m (aa0; m, -m, 0) \{\sigma_1\sigma_2\}^{am} \mathcal{Y}^{a,-m}_{f_1f_2}(\hat{p}'\hat{p})$$

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wracamy do str. 2 $\quad m_l \to \nu, \; m_{l'} \to \nu'$

$$W = \sum_\nu \sum_{\nu'} c(lsj; \nu, \mu-\nu, \mu)\, c(l's'j'; \nu', \mu'-\nu', \mu')$$

$$\sum_m (aa0; m, -m, 0)$$

$$\int d\hat{p}' \int d\hat{p}\, Y^*_{l\nu}(\hat{p})\, Y_{l'\nu'}(\hat{p}')\, \mathcal{Y}^{a,-m}_{f_1f_2}(\hat{p}'\hat{p})$$

$$Y_{l'\nu'}(\hat{p}') = (-1)^{\nu'} Y^*_{l',-\nu'}(\hat{p}')$$

$$W = \sum_\nu \sum_{\nu'} c(lsj; \nu, \mu-\nu, \mu)\, c(l's'j'; \nu', \mu'-\nu', \mu')$$

$$\sum_m (aa0; m, -m, 0)(-1)^{\nu'}$$

$$\sum_g (l'lg; -\nu', \nu, \nu-\nu')$$

$$\underbrace{\int d\hat{p}'\, d\hat{p}\, \mathcal{Y}^{*g\,\nu-\nu'}_{l'l}(\hat{p}'\hat{p})\, \mathcal{Y}^{a,-m}_{f_1f_2}(\hat{p}'\hat{p})}_{\delta_{l'f_1}\delta_{lf_2}\delta_{ga}\delta_{\nu-\nu',-m}} \qquad (10)$$

$$\langle s\, \mu-\nu | \{\sigma_1 \sigma_2\}^{am} | s'\, \mu'-\nu' \rangle$$
$$= (a\, s'\, s; m, \mu'-\nu', m+\mu'-\nu')$$
$$\delta_{\mu-\nu,\, m+\mu'-\nu'}\; 6\sqrt{\hat{a}\, \hat{s}'} \begin{Bmatrix} 1 & 1 & a \\ \frac{1}{2} & \frac{1}{2} & s' \\ \frac{1}{2} & \frac{1}{2} & s \end{Bmatrix} \quad (11)$$

$$\sum_{\nu \nu'} (l\, s\, j; \nu, \mu-\nu, \mu)(l'\, s'\, j'; \nu', \mu'-\nu', \mu')$$
$$\sum_{m} (a\, a\, 0; m, -m, 0)\,(-1)^{\nu'}\,(l'\, l\, a; -\nu', \nu, \nu-\nu')$$
$$\delta_{l' f_1}\, \delta_{l f_2}\, \delta_{\nu-\nu', -m}$$
$$(a\, s'\, s; m, \mu'-\nu', m+\mu'-\nu')\, \delta_{\mu-\nu,\, m+\mu'-\nu'}$$
$$6\sqrt{\hat{a}\, \hat{s}'} \begin{Bmatrix} 1 & 1 & a \\ \frac{1}{2} & \frac{1}{2} & s' \\ \frac{1}{2} & \frac{1}{2} & s \end{Bmatrix}$$

$$\nu - \nu' = -m \longrightarrow m = \nu' - \nu$$
$$\mu - \nu = m + \mu' - \nu'$$
$$\underbrace{\mu - \nu = \nu' - \nu + \mu' - \nu' = \mu' - \nu}$$
$$\Downarrow$$
$$\mu' = \mu$$

To dowodzi, że potencjał nie zmienia składowej z całkowitego momentu pędu układu dwóch nukleonów!

$$\langle p\,\alpha_2 | V_{1\pi} | p'\,\alpha_2' \rangle =$$

$$= -\frac{g_A^2}{4F_\pi^2} \langle t\, m_t | \vec{\tau}_1 \cdot \vec{\tau}_2 | t'\, m_t' \rangle$$

$$2\pi \sum_{a=0,2} (1\,1\,a;0\,0\,0) \sqrt{(2a+1)!}$$

$$\sum_{a_1+a_2=a} (p')^{a_1}\, p^{a_2} (-1)^{a_2} \frac{1}{\sqrt{(2a_1)!\,(2a_2)!}}$$

$$\sum_k \hat{k}\, (-1)^k\, g_{ka}$$

$$\sum_{f_1, f_2} \begin{Bmatrix} f_2 & f_1 & a \\ a_1 & a_2 & k \end{Bmatrix} (k\,a_1\,f_1;0\,0\,0)(k\,a_2\,f_2;0\,0\,0)$$

$$\delta_{l' f_1}\, \delta_{l f_2}\; 6\sqrt{\hat{a}\,\hat{s}'} \begin{Bmatrix} 1 & 1 & a \\ \tfrac{1}{2} & \tfrac{1}{2} & s' \\ \tfrac{1}{2} & \tfrac{1}{2} & s \end{Bmatrix}$$

$$S \left\{ \begin{array}{l} \displaystyle\sum_{\nu, \nu'} (l\,s\,j;\nu, \mu-\nu, \mu)(l'\,s'\,j';\nu', \mu'-\nu', \mu') \\ \displaystyle\sum_m (a, a, 0; m, -m, 0)\, (-1)^{\nu'} \\ (l'\,l\,a; -\nu', \nu, \nu-\nu')\; \delta_{\nu-\nu', -m} \\ (a\,s'\,s; m, \mu'-\nu', m+\mu'-\nu') \end{array} \right.$$

$$w(a,b,c,d;ef) = (-1)^{a+b+c+d} \begin{Bmatrix} a & b & e \\ d & c & f \end{Bmatrix}$$

$$(l' l a; -\nu', \nu, \nu-\nu') = (-1)^{l'+l-a}$$

$$(l l' a; \nu, -\nu', \nu-\nu') =$$

$$(l l' a; -\nu, \nu', \nu'-\nu) \underbrace{(-1)^{l'+l-a} (-1)^{l+l'-a}}_{1}$$

$$(l' l a; -\nu', \nu, \nu-\nu')(a s' s; m, \mu'-\nu', m+\mu'-\nu')$$

$$\overset{6.46}{=} (l l' a; -\nu, \nu', \nu'-\nu)(a s' s; m, \mu'-\nu', m+\mu'-\nu')$$

$$= \sum_{\omega} \sqrt{\hat{a}\,\hat{\omega}}\; W(l l' s s'; a \omega)$$
$$c(l' s' \omega; \nu', \mu'-\nu', \mu')$$
$$c(l \omega s; -\nu, \mu', \mu'-\nu)$$

$\mu = \mu'$

$$c(l \omega s; -\nu, \mu', \mu-\nu) =$$

$$= c(l \omega s; -\nu, \mu, \mu-\nu)$$

$$(-1)^{l+\nu} \sqrt{\frac{\hat{s}}{\hat{\omega}}}\, (l s \omega; -\nu, \nu-\mu, -\mu)$$

$$S = \sum_{\omega}{}' \sqrt{\hat{a}\hat{\omega}}\, w(l l' s s'; a \omega)$$

$$\sum_{\nu,\nu'}{}' (l s j; \nu, \mu-\nu, \mu)(l' s' j'; \nu', \mu-\nu', \mu)$$

$$\sum_{m}{}' (a a 0; m, -m, 0)(l' s' \omega; \nu', \mu-\nu', \mu)$$

$$(-1)^{l+\nu+\nu'} \sqrt{\frac{\hat{s}}{\hat{\omega}}}\, (l s \omega; \nu, \mu-\nu, \mu)$$

$$(-1)^{l+s-\omega}$$

$$\delta_{m, \nu'-\nu}$$

$$(a a 0; m, -m, 0) = (-1)^{a-m} \sqrt{\frac{1}{\hat{a}}}\, \underbrace{(a 0 a; m, 0, m)}_{1}$$

$$S = \sum_{\omega}{}' \sqrt{\hat{a}\hat{\omega}}\, w(l l' s s'; a \omega)$$

$$\sum_{\nu}{}' (l s j; \nu, \mu-\nu, \mu)(l s \omega; \nu, \mu-\nu, \mu)$$

$$\sum_{\nu'}{}' (l' s' j'; \nu', \mu'-\nu', \mu')(l' s' \omega; \nu', \mu'-\nu', \mu')$$

$$\sum_{m}{}' (-1)^{a-m} \frac{1}{\sqrt{\hat{a}}} (-1)^{l+\nu+\nu'} \sqrt{\frac{\hat{s}}{\hat{\omega}}}\, (-1)^{l+s-\omega}$$

$$= \sum_{\omega}{}' \sqrt{\hat{a}\hat{\omega}}\, w(l l' s s'; a \omega)\, \delta_{\omega j}\, \delta_{\omega j'}$$

$$\frac{1}{\sqrt{\hat{a}}} (-1)^{a} \sqrt{\frac{\hat{s}}{\hat{\omega}}}\, (-1)^{s-\omega}$$

$$(-1)^{\nu+\nu'-m} = (-1)^{\nu+\nu'-\nu'+\nu} = 1$$

$$S = \delta_{\mu\mu'}\delta_{jj'}(-1)^a \sqrt{\hat{s}}\,(-1)^{s-j}$$

$$(-1)^{l+l'+s+s'} \begin{Bmatrix} l & l' & a \\ s' & s & j \end{Bmatrix} =$$

$$= \delta_{jj'}\delta_{\mu\mu'}\sqrt{\hat{s}} \begin{Bmatrix} l & l' & a \\ s' & s & j \end{Bmatrix} (-1)^{j+l+l'+s'}$$

(12)

$$\langle p\alpha_2 | V_{1\pi} | p'\alpha' \rangle =$$

$$-\frac{g_A^2}{4F_\pi^2} \langle t m_t | \vec{\tau}_1 \cdot \vec{\tau}_2 | t' m_t' \rangle$$

$$\underline{2\pi} \sum_{a=0,2}{}' \underline{(11a;000)}\, \sqrt{(2a+1)!}$$

$$\sum_{a_1+a_2=a}{}' (p')^{a_1} p^{a_2} (-1)^{a_2} \frac{1}{\sqrt{(2a_1)!(2a_2)!}}$$

$$\sum_{k}{}' \hat{k}\,(-1)^k g_{ka}$$

$$\begin{Bmatrix} l & l' & a \\ a_1 & a_2 & k \end{Bmatrix} (k a_1 l'; 000)(k a_2 l; 000)$$

$$\underline{6}\,\sqrt{\hat{a}\,\hat{s}'} \begin{Bmatrix} 1 & 1 & a \\ \frac{1}{2} & \frac{1}{2} & s' \\ \frac{1}{2} & \frac{1}{2} & s \end{Bmatrix} \sqrt{\hat{s}} \begin{Bmatrix} l & l' & a \\ s' & s & j \end{Bmatrix} (-1)^{j+l+l'+s'}$$

$$\delta_{jj'}\delta_{\mu\mu'}$$

$$\langle t m_t | \vec{\tau}_1 \cdot \vec{\tau}_2 | t' m_t' \rangle =$$
$$= \delta_{tt'} \delta_{m_t m_t'} \left[2(t+1)t - 3 \right] \qquad (13)$$
$$\langle t m_t | \mathbb{1}^{iso} | t' m_t' \rangle = \delta_{tt'} \delta_{m_t m_t'}$$

Końcowy wzór z uwzględnieniem "regulatorów"

$$\langle p \alpha_2 | V_{1\pi} | p' \alpha_2' \rangle = \delta_{jj'} \delta_{\mu\mu'} \delta_{tt'} \delta_{m_t m_t'}$$
$$\left(- \frac{g_A^2}{4 F_\pi^2} \right) f_\Lambda(p) f_\Lambda(p') \left[2(t+1)t - 3 \right]$$
$$12\pi \sqrt{\hat{s} \hat{s}'} (-1)^{j+s'+l+l'}$$
$$\sum_{a=0,2} \sqrt{\hat{a}} \, (1 1 a ; 000) \sqrt{(2a+1)!} \begin{Bmatrix} l & l' & a \\ s' & s & j \end{Bmatrix}$$
$$\begin{Bmatrix} 1 & 1 & a \\ \frac{1}{2} & \frac{1}{2} & s \\ \frac{1}{2} & \frac{1}{2} & s' \end{Bmatrix}$$
$$\sum_{a_1 + a_2 = a} (p')^{a_1} p^{a_2} (-1)^{a_2} \frac{1}{\sqrt{(2a_1)! \, (2a_2)!}}$$
$$\sum_k \hat{k} \, (-1)^k g_{ka} \begin{Bmatrix} l & l' & a \\ a_1 & a_2 & k \end{Bmatrix} (k a_1 l' ; 000)$$
$$(k a_2 l ; 000) ,$$

gdzie
$$g_{ka} = \int_{-1}^{1} dx \, P_k(x) \frac{|\vec{q}|^{2-a}}{m_\pi^2 + \vec{q}^{\,2}} ,$$
$$|\vec{q}| = \sqrt{(p')^2 + p^2 - 2pp'x}$$

Zachowanie parzystości można pokazać, korzystając z właściwości współczynników Clebscha-Gordana

$c(k a_1 l'; 000) \neq 0$, jeśli $k + a_1 + l'$ - parzyste

$c(k a_2 l; 000) \neq 0$, jeśli $k + a_2 + l$ - parzyste

$$(-1)^{k+a_1+l'} = 1 \Rightarrow (-1)^{l'} = (-1)^{k+a_1} =$$

$$= (-1)^{k+a_1+k+a_2+l} = (-1)^{a_1+a_2+l} =$$

$$= (-1)^{a} (-1)^{l} = (-1)^{l},$$ bo a musi być parzyste ze względu na $c(11a; 000)$

Dlatego $(-1)^{l'} = (-1)^{l}$

Zachowanie spinu ($s = s'$) wynika z właściwości symbolu $9j$

$$\begin{Bmatrix} 1 & 1 & a \\ \frac{1}{2} & \frac{1}{2} & s' \\ \frac{1}{2} & \frac{1}{2} & s \end{Bmatrix} \neq 0 \Rightarrow$$

1. $a = 0$, co daje $s = s'$
2. $a = 2$ jest możliwe tylko wtedy, gdy $s = s' = 1$

w trzeciej kolumnie mamy takie same reguły jak dla j_1, j_2 i j we współczynniku Clebscha-Gordana

Składowe kontaktowe w V^{LO}

$$\langle \vec{p} \,|V_{ct}|\, \vec{p}\,' \rangle = C_S\, \mathbb{1}^{sp} \otimes \mathbb{1}^{iso} + C_T\, \vec{\sigma}_1 \cdot \vec{\sigma}_2\, \mathbb{1}^{iso}$$

Wracamy do strony 6

$$\langle p \alpha_2 |V_{ct}| p' \alpha_2' \rangle = \text{(włączając regulatory)}$$

$$= f_\Lambda(p)\, f_\Lambda(p')$$

$$\int d\hat{p} \int d\hat{p}\,' \sum_{m_l} \sum_{m_l'} c(l s j; m_l, \mu - m_l, \mu)$$

$$c(l' s' j'; m_l', \mu' - m_l', \mu')$$

$$Y^*_{l m_l}(\hat{p})\, Y_{l' m_l'}(\hat{p}\,')$$

$$\underbrace{\langle t m_t | \langle s \mu - m_l | \langle \vec{p} \,|V_{ct}|\, \vec{p}\,' \rangle | s' \mu' - m_l' \rangle | t' m_t' \rangle}$$

$$\delta_{tt'}\, \delta_{m_t m_t'}\, \delta_{ss'}\, \delta_{\mu - m_l, \mu' - m_l'}$$

$$[C_S + C_T (2s(s+1) - 3]$$

$$\left.\begin{aligned} Y_{00}(\hat{p}) &= \frac{1}{\sqrt{4\pi}} \\ Y_{00}(\hat{p}\,') &= \frac{1}{\sqrt{4\pi}} \end{aligned}\right\} \text{stałe!}$$

$$\langle p\alpha_2 | V_{ct} | p'\alpha_{2'} \rangle =$$

$$= f_\Lambda(p) f_\Lambda(p') \delta_{tt'} \delta_{m_t m_{t'}} \delta_{ss'}$$

$$\left[c_S + c_T \left(2s(s+1) - 3 \right) \right]$$

$$\sum_{m_l} c(l s j; m_l, \mu - m_l, \mu)$$

$$\sum_{m_{l'}} c(l' s' j'; m_{l'}, \mu' - m_{l'}, \mu') \, \delta_{\mu - m_l, \mu' - m_{l'}}$$

$$4\pi \underbrace{\int d\hat{p} \, Y^*_{l m_l}(\hat{p}) \, Y_{00}(\hat{p})}_{\delta_{l0} \delta_{m_l 0}}$$

$$\underbrace{\int d\hat{p}' \, Y_{00}(\hat{p}') \, Y_{l' m_{l'}}(\hat{p}')}_{\delta_{l'0} \delta_{m_{l'} 0}}$$

$$= f_\Lambda(p) f_\Lambda(p') \delta_{tt'} \delta_{m_t m_{t'}} \, 4\pi$$

$$\left[c_S + c_T \left(2s(s+1) - 3 \right) \right] \delta_{\mu\mu'}$$

$$\underbrace{\delta_{sj} \, \delta_{s'j'} \, \delta_{ss'}}_{j = j' = s = s'} \; \delta_{l'0} \, \delta_{l0}$$

Dodatek: spinowe elementy macierzowe

1. Definicja zredukowanego elementu macierzowego

$$\langle L'\nu' | A_{kq} | L\nu \rangle \equiv c(kLL'; q\nu\nu') \underbrace{\langle L' \| A_k \| L \rangle}$$

nie zależy od liczb "magnetycznych" q, ν i ν'

Przykład:

$\langle \frac{1}{2} m' | \sigma_{1q} | \frac{1}{2} m \rangle$, gdzie

σ_{1q} to składowa sferyczna $\vec{\sigma}$ (podwojonego operatora spinu)

$$\langle \frac{1}{2} m' | \sigma_{1q} | \frac{1}{2} m \rangle = c(1 \tfrac{1}{2} \tfrac{1}{2}; q\, m\, m') \langle \tfrac{1}{2} \| \sigma_1 \| \tfrac{1}{2} \rangle$$

Jak dostać $\langle \frac{1}{2} \| \sigma_1 \| \frac{1}{2} \rangle$?

Rozważamy najprostszy przypadek:

$$\langle \tfrac{1}{2} \tfrac{1}{2} | \sigma_z | \tfrac{1}{2} \tfrac{1}{2} \rangle = \langle \tfrac{1}{2} \tfrac{1}{2} | \sigma_{10} | \tfrac{1}{2} \tfrac{1}{2} \rangle =$$

$$= \underbrace{c(1 \tfrac{1}{2} \tfrac{1}{2}; 0 \tfrac{1}{2} \tfrac{1}{2})}_{-\frac{1}{\sqrt{3}}} \langle \tfrac{1}{2} \| \sigma_1 \| \tfrac{1}{2} \rangle = 1$$

Dlatego $\langle \frac{1}{2} \| \sigma_1 \| \frac{1}{2} \rangle = -\sqrt{3}$

$$H \equiv \langle (\overset{1}{\tfrac{1}{2}}\overset{2}{\tfrac{1}{2}}) s'\mu' | \{\sigma(1)\sigma(2)\}^{km} | (\overset{1}{\tfrac{1}{2}}\overset{2}{\tfrac{1}{2}}) s\mu \rangle$$

$$= \langle (\tfrac{1}{2}\tfrac{1}{2}) s'\mu' | \{ \{\overset{j1}{\sigma(1)}\overset{j2}{\sigma(2)}\}^{\overset{j12}{k}} | (\overset{j3}{\tfrac{1}{2}}\overset{j4}{\tfrac{1}{2}}) \overset{j34}{s} \rangle \}^{s'\mu'}$$

$$c(k s s'; m, \mu, \mu')$$

$$= c(k s s'; m, \mu, \mu') \sum_{\substack{s_1, s_2 \\ j13 \; j24}} \sqrt{\hat{k}\hat{s}\hat{s}_1\hat{s}_2} \begin{Bmatrix} 1 & 1 & k \\ \tfrac{1}{2} & \tfrac{1}{2} & s \\ s_1 & s_2 & s' \end{Bmatrix}$$

$$\langle (\tfrac{1}{2}\tfrac{1}{2}) s'\mu' | \{ \{\sigma(1)|\tfrac{1}{2}\rangle\}^{s_1} \{\sigma(2)|\tfrac{1}{2}\rangle\}^{s_2} \}^{s'\mu'}$$

$$= c(k s s'; m, \mu, \mu')$$

$$\sum_{s_1, s_2} \sqrt{\hat{k}\hat{s}\hat{s}_1\hat{s}_2} \begin{Bmatrix} 1 & 1 & k \\ \tfrac{1}{2} & \tfrac{1}{2} & s \\ s_1 & s_2 & s' \end{Bmatrix}$$

$$\sum_{\nu'} (\tfrac{1}{2}\tfrac{1}{2}s'; \nu', \mu'-\nu', \mu') \sum_{\nu} (s_1 s_2 s'; \nu, \mu'-\nu, \mu')$$

$$\underbrace{\langle \tfrac{1}{2}\nu' | \{\sigma(1)|\tfrac{1}{2}\rangle\}^{s_1}_{\nu}}_{?}$$

$$\underbrace{\langle \tfrac{1}{2}\mu'-\nu' | \{\sigma(2)|\tfrac{1}{2}\rangle\}^{s_2}_{\mu'-\nu}}_{?}$$

$$\langle \tfrac{1}{2}\,\nu' | \{\sigma(1)\,|\tfrac{1}{2}\rangle\}^{s_1 \nu} =$$

$$= \sum_{\alpha} (1\,\tfrac{1}{2}\,s_1; \alpha, \nu-\alpha, \nu)\; \underbrace{\langle \tfrac{1}{2}\,\nu' | \sigma(1)_\alpha | \tfrac{1}{2}\,\nu-\alpha\rangle}_{C(1\,\frac{1}{2}\,\frac{1}{2};\, \alpha, \nu-\alpha, \nu')\,\langle \frac{1}{2} \| \sigma_1 \| \frac{1}{2}\rangle}$$

$$= -\sqrt{3}\, \underbrace{\sum_{\alpha} (1\,\tfrac{1}{2}\,s_1; \alpha, \nu-\alpha, \nu)\,(1\,\tfrac{1}{2}\,\tfrac{1}{2}; \alpha, \nu-\alpha, \nu')}_{\delta_{s_1 \frac{1}{2}}\,\delta_{\nu'\nu}}$$

$$= -\sqrt{3}\; \delta_{s_1 \frac{1}{2}}\, \delta_{\nu'\nu}$$

$$\langle \tfrac{1}{2}\,\mu'-\nu' | \{\sigma(2)\,|\tfrac{1}{2}\rangle\}^{s_2\, \mu'-\nu'} =$$

$$= -\sqrt{3}\; \delta_{s_2 \frac{1}{2}}\, \delta_{\nu'\nu}$$

$$H = c(k s s'; m, \mu, \mu') \sqrt{\hat{k}\,\hat{s}\,\hat{\tfrac{1}{2}}\,\hat{\tfrac{1}{2}}} \begin{Bmatrix} 1 & 1 & k \\ \frac{1}{2} & \frac{1}{2} & s \\ \frac{1}{2} & \frac{1}{2} & s' \end{Bmatrix} (-\sqrt{3})(-\sqrt{3})$$

$$\underbrace{\sum_{\nu} \left(\tfrac{1}{2}\,\tfrac{1}{2}\,s'; \nu, \mu'-\nu, \mu'\right)\left(\tfrac{1}{2}\,\tfrac{1}{2}\,s'; \nu, \mu'-\nu, \mu'\right)}_{1}$$

$$\hat{\tfrac{1}{2}} = 2\cdot\tfrac{1}{2}+1 = 2$$

$$H = c(k s s'; m, \mu, \mu')\, 6\sqrt{\hat{k}\,\hat{s}} \begin{Bmatrix} 1 & 1 & k \\ \frac{1}{2} & \frac{1}{2} & s \\ \frac{1}{2} & \frac{1}{2} & s' \end{Bmatrix}$$