Wykład 12 W2FT

wykorzystanie reguł Feynmana

An Introduction to QED and QCD

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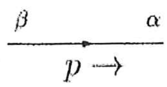
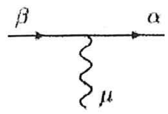
http://www.hep.man.ac.uk/u/forshaw/NorthWest/QED.pdf

For every ...	draw ...	write ...
Internal photon line		$\dfrac{-ig^{\mu\nu}}{q^2+i\epsilon}$
Internal fermion line		$\dfrac{i(\not{p}+m)_{\alpha\beta}}{p^2-m^2+i\epsilon}$
Vertex		$-ie\gamma^{\mu}_{\alpha\beta}$
Outgoing electron		$\overline{u}_s(p)$
Incoming electron		$u_s(p)$
Outgoing positron		$v_s(p)$
Incoming positron		$\overline{v}_s(p)$
Outgoing photon		$\epsilon^{*\mu}$
Incoming photon		ϵ^{μ}

- Attach a directed momentum to every internal line
- Conserve momentum at every vertex

Table 4.1 Feynman rules for QED. μ, ν are Lorentz indices and α, β are spinor indices.

zastosowanie reguł Feynmana
do przykładowych procesów QED
w najniższym rzędzie rachunku zaburzeń

Ogólny zapis różniczkowego przekroju czynnego $1+2 \to 1'+2'$

$$d\sigma = \frac{1}{|\vec{v}_1 - \vec{v}_2|} \frac{1}{2E_1} \frac{1}{2E_2} |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_1+p_2-k_1-k_2)$$

$$\frac{d^3k_1}{2\omega_1(2\pi)^3} \frac{d^3k_2}{2\omega_2(2\pi)^3} \quad S \qquad (1)$$

S potrzebny tylko wtedy, gdy mamy cząstki identyczne w kanale końcowym

$$p_1+p_2 = k_1+k_2$$

$$E_1 \equiv p_1^0 = \sqrt{m_1^2 + \vec{p}_1^{\,2}}$$

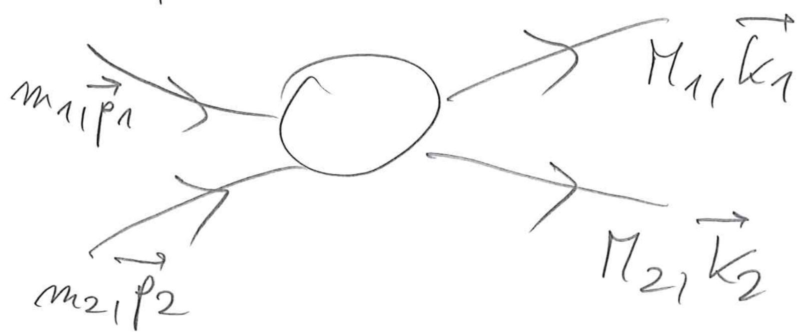
$$E_2 \equiv p_2^0 = \sqrt{m_2^2 + \vec{p}_2^{\,2}}$$

$$\omega_1 \equiv k_1^0 \equiv \sqrt{M_1^2 + \vec{k}_1^{\,2}}$$

$$\omega_2 = k_2^0 \equiv \sqrt{M_2^2 + \vec{k}_2^{\,2}}$$

$$E_1 + E_2 = \omega_1 + \omega_2$$

$$\vec{p}_1 + \vec{p}_2 = \vec{k}_1 + \vec{k}_2$$



Cała dynamika jest ukryta w $|M|^2$
Reszta to zależności kinematyczne!

Uwaga:
wzór (1) musi być spójny
ze sposobem normalizacji spinorów
$u(p,s)$ i $v(p,s)$, które występują
w regułach Feynmana
w szczególności

$$u(p,s) = \sqrt{E+m}\begin{pmatrix} \chi_s \\ \frac{\vec{p}\cdot\vec{\sigma}}{E+m}\chi_s \end{pmatrix},$$

gdzie $E = \sqrt{m^2+\vec{p}^{\,2}}$, $\chi_s^\dagger \chi_s = 1$

χ_s spełnia
$(\hat{s}_r\cdot\vec{\sigma})\chi_s = \chi_s$, gdzie
$\hat{s}_r = \hat{s}_r(\Theta,\phi) = (\sin\Theta\cos\phi, \sin\Theta\sin\phi, \cos\Theta)$

W ogólnym przypadku możemy mieć do czynienia ze spolaryzowanymi leptonami w stanie początkowym lub końcowym. Wówczas albo używamy spinorów Diraca dla określonej helicity, albo musimy określić czterowektor s^μ, który dostajemy przez transformację Lorentza czterowektora $s^\mu_{rest} \equiv (0, \hat{s}_r)$ z układu, w którym cząstka spoczywa do układu, w którym pęd cząstki wynosi $\vec{p}$.
Wykorzystując wzór (11.19) z książki J. D. Jacksona "Elektrodynamika klasyczna", dostajemy

$$\vec{\beta} = -\frac{\vec{p}}{E}, \quad \gamma \equiv (1-\vec{\beta}^2)^{-\frac{1}{2}} = \frac{E}{m}$$

$$s^0 = \gamma\left(0 + \frac{\vec{p}\cdot\hat{s}_r}{E}\right) = \frac{\vec{p}\cdot\hat{s}_r}{m}$$

$$\vec{s} = \hat{s}_r + \frac{(\gamma-1)}{\beta^2}\,\frac{\vec{p}\cdot\hat{s}_r}{E}\,\frac{\vec{p}}{E} =$$

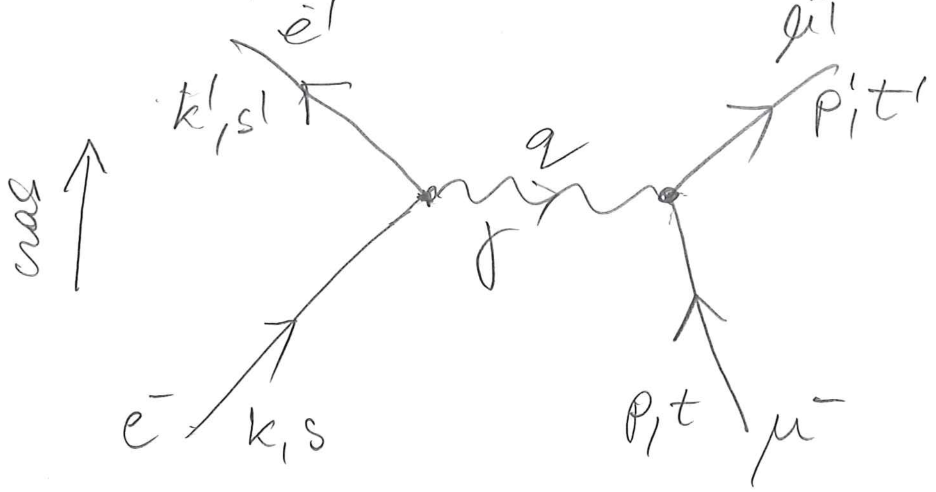
$$= \hat{s}_r + \frac{(E-m)\,E^2}{m\,\vec{p}^{\,2}E^2}(\vec{p}\cdot\hat{s}_r)\,\vec{p} =$$

$$= \hat{s}_r + \frac{(E-m)(\vec{p}\cdot\hat{s}_r)\vec{p}}{m(E-m)(E+m)} =$$

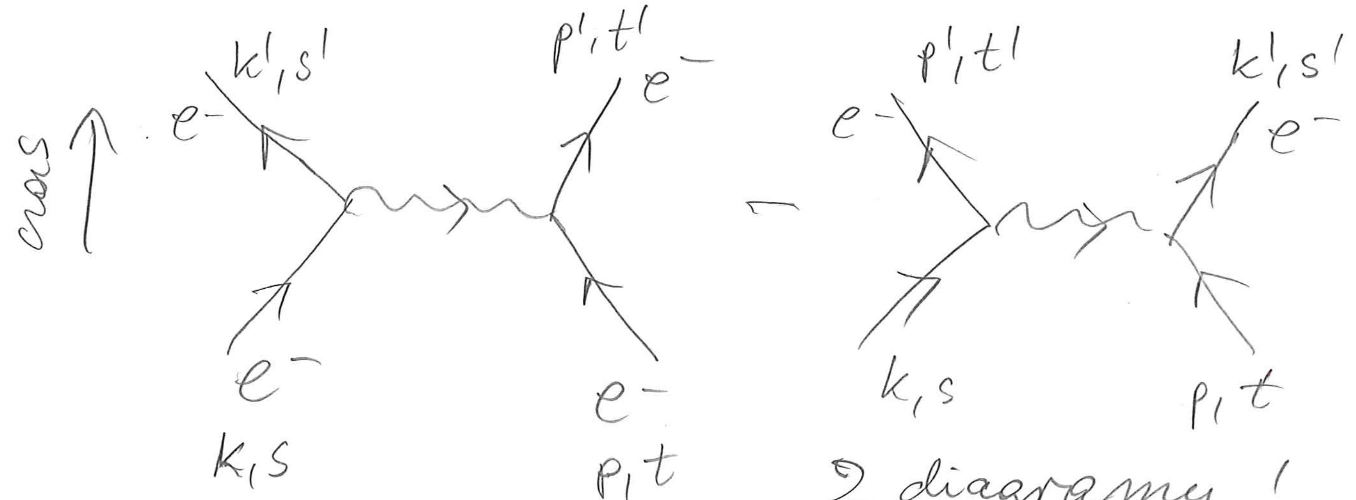
$$= \hat{s}_r + \frac{(\vec{p}\cdot\hat{s}_r)\vec{p}}{m(E+m)} = s^\mu$$

Zaczynamy od elastycznego rozpraszania elektron-mion

Dlaczego? Bo jest prostsze niż rozpraszanie elektron-elektron!



1 diagram



2 diagramy!

$$k = k' + q \quad \text{lub} \quad p' = q + p$$

$$\mathcal{M} = \bar{u}(k',s')(-ie\gamma_\alpha)u(k,s)$$
$$\bar{u}(p',t')(-ie\gamma_\beta)u(p,t)$$
$$\frac{(-ig^{\alpha\beta})}{q^2+i\varepsilon} \qquad \boxed{e^2 = 4\pi\alpha \approx \frac{4\pi}{137}}$$

$$q^2 = (k-k')^2 = (k^0-k'^0)^2 - (\vec{k}-\vec{k}')^2 =$$
$$= k^{02} + k'^{02} - 2k^0k'^0 - \vec{k}^2 - \vec{k}'^2 + 2\vec{k}\cdot\vec{k}'$$
$$= 2m_e^2 - 2\sqrt{m_e^2+\vec{k}^2}\sqrt{m_e^2+\vec{k}'^2} + 2\vec{k}\cdot\vec{k}'$$

$$|\mathcal{M}|^2 = \mathcal{M}\mathcal{M}^*$$

Rozważmy część elektronową $|\mathcal{M}|^2$

$$E_{\alpha\beta} \equiv \left(\bar{u}(k',s')\gamma_\alpha u(k,s)\right)^*$$
$$\bar{u}(k',s')\gamma_\beta u(k,s)$$

Trick: $\left(\bar{u}(k',s')\gamma_\alpha u(k,s)\right)^* =$

$$= \left(\bar{u}(k',s')\gamma_\alpha u(k,s)\right)^\dagger =$$
$$= u(k,s)^\dagger \gamma_\alpha^\dagger \bar{u}(k',s')^\dagger = \begin{pmatrix} \bar{u} = u^\dagger\gamma^0 \\ \gamma^0 = (\gamma^0)^\dagger \\ (\gamma^0)^2 = \mathbb{1} \end{pmatrix}$$

$$= \underbrace{u(k,s)^\dagger\gamma^0}_{\bar{u}(k,s)}\underbrace{\gamma^0\gamma_\alpha^\dagger\gamma^0}_{\gamma_\alpha} u(k',s') =$$

$$= \bar{u}(k,s)\gamma_\alpha u(k',s')$$

$$E_{\alpha\beta} = \bar{u}(k,s)\gamma_\alpha u(k',s')\bar{u}(k',s')\gamma_\beta u(k,s)$$

zapisując jawnie indeksy macierzy, dostajemy:

$$E_{\alpha\beta} = \sum_{a,b}\sum_{c,d} \bar{u}(k,s)_a (\gamma_\alpha)_{ab}\, u(k',s')_b\, \bar{u}(k',s')_c (\gamma_\beta)_{cd}\, u(k,s)_d =$$

$$= \sum_{a,b,c,d} u(k,s)_d\, \bar{u}(k,s)_a (\gamma_\alpha)_{ab}\, u(k',s')_b\, \bar{u}(k',s')_c (\gamma_\beta)_{cd}$$

$$u(k,s)_d\, \bar{u}(k,s)_a = \big(\underbrace{u(k,s)\bar{u}(k,s)}_{\text{macierz } 4\times 4}\big)_{da}$$

$$u(k',s')_b\, \bar{u}(k',s')_c = \big(\underbrace{u(k',s')\bar{u}(k',s')}_{\text{macierz } 4\times 4}\big)_{bc}$$

Dodatkowo

$$u(k,s)\bar{u}(k,s) = \frac{1}{2}(\not{k} + m_e)(1 + \gamma_5 \not{s})$$

$$u(k',s')\bar{u}(k',s') = \frac{1}{2}(\not{k}' + m_e)(1 + \gamma_5 \not{s}'),$$

$$\not{s} \equiv s^\nu \gamma_\nu = s^0\gamma^0 - s^1\gamma^1 - s^2\gamma^2 - s^3\gamma^3$$

$$\not{s}' = s'^\nu \gamma_\nu = s'^0\gamma^0 - s'^1\gamma^1 - s'^2\gamma^2 - s'^3\gamma^3$$

W tym właśnie miejscu potrzebujemy (przetransformowanych) czterowektorów s^μ ze strony 4 !

$$E_{\alpha\beta} = \sum_{a,b,c,d} \frac{1}{2}\left[(\not{k}+m_e)(1+\gamma_5 \not{s})\right]_{da}$$

$$(\gamma_\alpha)_{ab}\ \frac{1}{2}\left[(\not{k}'+m_e)(1+\gamma_5 \not{s}')\right]_{bc} (\gamma_\beta)_{cd} =$$

$$= \frac{1}{4} \mathrm{Tr}\Big((\not{k}+m_e)(1+\gamma_5\not{s})\gamma_\alpha$$

$$(\not{k}'+m_e)(1+\gamma_5\not{s}')\gamma_\beta\Big)$$

Jeśli spin wylatującego elektronu nie jest mierzony

$$E_{\alpha\beta} \to \sum_{s'} E_{\alpha\beta} =$$

$$= \frac{1}{2}\mathrm{Tr}\Big((\not{k}+m_e)(1+\gamma_5\not{s})\gamma_\alpha(\not{k}'+m_e)\gamma_\beta\Big),$$

ponieważ

$$\sum_{s'} u(k',s')\bar{u}(k',s') = (\not{k}'+m_e)$$

Uwaga na zapis!

$$1+\gamma_5 \not{s} \equiv \mathbb{1}_{4\times4} + \gamma_5 \not{s}$$

macierz identycznościowa 4×4

$$\not{k} + m_e \equiv \not{k} + m_e \mathbb{1}_{4\times4}$$

Można pokazać (tw. o śladach, Mathematica), że

$$\sum_{s'} E_{\alpha\beta} = 2\left(k'_\alpha k_\beta + k_\alpha k'_\beta + m_e^2 g_{\alpha\beta} - (k\cdot k')g_{\alpha\beta} + i m_e \varepsilon_{\beta\alpha\gamma\delta} s^\gamma (k-k')^\delta\right),$$

gdzie $\varepsilon^{0123} = +1$, $\varepsilon_{0123} = -1$

Jeśli dodatkowo electron w stanie początkowym jest niespolaryzowany, wówczas

$$\sum_{s'} E_{\alpha\beta} \to \frac{1}{2}\sum_{s,s'} E_{\alpha\beta} \equiv \overline{E}_{\alpha\beta} =$$

$$= \frac{1}{2} Tr\left((\not{k}+m_e)\gamma_\alpha(\not{k}'+m_e)\gamma_\beta\right) = \ldots =$$

$$= 2\left(k'_\alpha k_\beta + k_\alpha k'_\beta + m_e^2 g_{\alpha\beta} - (k\cdot k')g_{\alpha\beta}\right)$$

Dokładnie tak samo postępujemy
dla części mionowej (górne indeksy)

$$M^{\alpha\beta} = (\bar{u}(p',t')\gamma^{\alpha}u(p,t))^{*}\,\bar{u}(p',t')\gamma^{\beta}u(p,t)$$

Możemy rozpatrywać w pełni spolaryzowany
przypadek (z określonymi t i t') $M^{\alpha\beta}$,

$$\sum_{t'} M^{\alpha\beta} \text{ lub } \frac{1}{2}\sum_{t,t'} M^{\alpha\beta} \equiv \overline{M}^{\alpha\beta} =$$

$$= 2\left(p'^{\alpha}p^{\beta} + p^{\alpha}p'^{\beta} + M^2 g^{\alpha\beta} - (p\cdot p')g^{\alpha\beta}\right),$$

gdzie $M = m_{\mu}$ (masa mionu)

W dalszym ciągu pokażę szczegółowe
przeliczenia dla kompletnie
niespolaryzowanego przypadku,
ale zawsze można wrócić
do ogólnych wyrażeń na $E_{\alpha\beta}$ i $M^{\alpha\beta}$.

$$|\overline{\mathcal{M}}|^2 = \overline{E}_{\alpha\beta}\,\overline{M}^{\alpha\beta}\left(\frac{e^2}{q^2}\right)^2$$

$$\overline{M}^{\alpha\beta} = 2\left(p'^{\alpha}p^{\beta} + p^{\alpha}p'^{\beta} + (M^2 - (p\cdot p')g^{\alpha\beta}\right)$$

$$\overline{E}_{\alpha\beta}\overline{M}^{\alpha\beta} = 4\left(k'_{\alpha}k_{\beta} + k_{\alpha}k'_{\beta} + (m^2 - (k\cdot k')g_{\alpha\beta}\right)$$
$$\left(p'^{\alpha}p^{\beta} + p^{\alpha}p'^{\beta} + (M^2 - (p\cdot p')g^{\alpha\beta}\right)$$

$$= 4\Big((k'\cdot p')(k\cdot p) + (k'\cdot p)(k\cdot p')$$
$$+ (M^2 - (p\cdot p'))(k\cdot k')$$
$$+ (k\cdot p')(k'\cdot p) + (k\cdot p)(k'\cdot p')$$
$$+ (M^2 - (p\cdot p')(k\cdot k')$$
$$+ (m^2 - (k\cdot k'))(p\cdot p')$$
$$+ (m^2 - (k\cdot k'))(p\cdot p')$$
$$+ (m^2 - (k\cdot k'))(M^2 - (p\cdot p'))\underbrace{g_{\alpha\beta}g^{\alpha\beta}}_{4?}\Big)$$

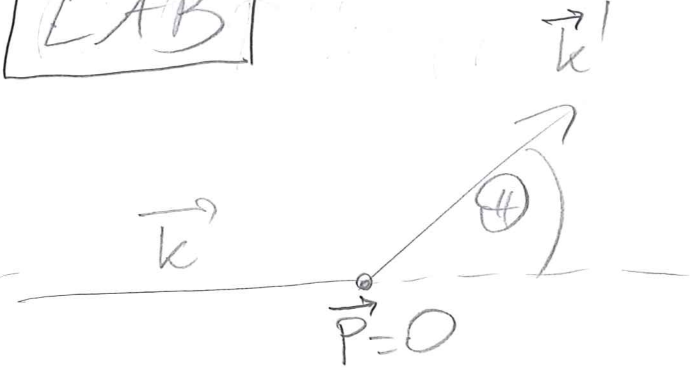
$$= 4\Big(2(k'\cdot p')(k\cdot p) + 2(k'\cdot p)(k\cdot p')$$
$$+ 2(M^2 - (p\cdot p'))(k\cdot k')$$
$$+ 2(m^2 - (k\cdot k'))(p\cdot p')$$
$$+ 4(m^2 - (k\cdot k'))(M^2 - (p\cdot p'))\Big)$$

$$
\begin{aligned}
= 4 \Big(& 2(k'\cdot p')(k\cdot p) \\
& + 2(k'\cdot p)(k\cdot p') \\
& + 2M^2(k\cdot k') \\
& - 2(p\cdot p')(k\cdot k') \\
& + 2m^2(p\cdot p') \\
& - 2(k\cdot k')(p\cdot p') \\
& + 4m^2M^2 - 4m^2(p\cdot p') \\
& - 4M^2(k\cdot k') \\
& + 4(k\cdot k')(p\cdot p') \Big)
\end{aligned}
$$

$$
\begin{aligned}
= 4 \Big(& 2(k\cdot p)(k'\cdot p') \\
& + 2(k'\cdot p)(k\cdot p') \\
& - 2M^2(k\cdot k') \\
& - 2m^2(p\cdot p') \\
& + 4m^2M^2 \Big) =
\end{aligned}
$$

$$
\begin{aligned}
= 8 \Big(& (k\cdot p)(k'\cdot p') \\
& + (k'\cdot p)(k\cdot p') \\
& - M^2(k\cdot k') \\
& - m^2(p\cdot p') + 2m^2M^2 \Big)
\end{aligned}
$$

LAB



$m \equiv m_e$
$M \equiv m_\mu$

$$\sqrt{m^2+k^2}+M=\sqrt{m^2+\vec{k}'^2}+\sqrt{M^2+\vec{P}'^2}$$

$$\vec{k}=\vec{k}'+\vec{P}' \Rightarrow \vec{P}'=\vec{k}-\vec{k}'$$

$$\sqrt{m^2+k^2}+M=\sqrt{m^2+k'^2}+\sqrt{M^2+k^2+k'^2-2kk'\cos\Theta}$$

Dla ustalonego Θ jest to równanie

na $k' \equiv |\vec{k}'|$

$$M=\sqrt{m^2+k'^2}+\sqrt{A+k'^2-Bk'}$$

$$k^2+m^2+2\sqrt{k^2+m^2}\,M+M^2=\left(M+\sqrt{k^2+m^2}\right)^2$$

$$k' \overset{M}{=} \frac{k\left(\left(m^2+\sqrt{k^2+M^2}\,M\right)\cos\Theta+\left(M+\sqrt{k^2+m^2}\right)\sqrt{M^2+m^2\cos^2\Theta-m^2}\right)}{m^2+M\left(2\sqrt{k^2+m^2}+M\right)+k^2\sin^2\Theta}$$

$$k'(m=0)=\frac{kM}{k+M-k\cos\Theta}$$

$$k' \cdot p' = k'(k + P - k') =$$
$$= k' \cdot k + k' \cdot P - k' \cdot k' =$$
$$= k_0 k_0' - k k' \cos \textcircled{\text{II}} + k_0' M - m^2$$

$$k \cdot P = k_0 M$$
$$k' \cdot P = k_0' M$$
$$k \cdot p' = k \cdot (k + P - k') = k \cdot k + k \cdot P - k \cdot k'$$
$$= m^2 + k_0 M - k_0 k_0' + k k' \cos \textcircled{4}$$
$$k \cdot k' = k_0 k_0' - k k' \cos \textcircled{4}$$
$$P \cdot P' = P(k + P - k') = k \cdot P + P \cdot P - k' \cdot P$$
$$= k_0 M + M^2 - k_0' M$$

$$\sum_{\substack{s, s' \\ t, t'}} |\mathcal{M}|^2 \stackrel{!}{=} 8 \Big(k_0 M (k' (k + P - k')) + k_0' M (k \cdot (k + P - k')) - (k \cdot k') M^2 - m^2 M (k_0 + M - k_0') + 2 m^2 M^2 \Big)$$

$$e^2 = 4\pi\alpha$$

$$= 8\Big(k_0 M (k\cdot k' + M k_0' - m^2)$$
$$+ k_0' M (m^2 + M k_0 - k\cdot k')$$
$$- M^2 k\cdot k' - m^2 M k_0 - m^2 M^2$$
$$+ m^2 M k_0' + 2 m^2 M^2 \Big)$$

$$= 8\Big(-(k\cdot k')(-k_0 M + k_0' M + M^2)$$
$$+ m^2 M^2$$
$$+ M^2 k_0 k_0' - m^2 M k_0$$
$$+ m^2 M k_0' + M^2 k_0 k_0' - m^2 M k_0$$
$$+ m^2 M k_0' \Big)$$

$$= 8\Big(-(k\cdot k')\big(M^2 + (k_0' - k_0) M\big)$$
$$+ 2 M^2 k_0 k_0' + 2 m^2 M (k_0' - k_0)$$
$$+ m^2 M^2 \Big) \equiv A$$

Czynnik przestrzeni fazowej

$$\rho = d^3k' d^3p' \delta^4(k+P-k'-P') =$$

$$= d^3k' d^3P' \delta(k_0+M-k_0'-P_0')$$

$$\delta^3(\vec{k}-\vec{k}'-\vec{P}')$$

$$= d^3k' \delta(k_0+M-k_0'-\tilde{P}_0')$$

$$\tilde{P}_0' = \sqrt{M^2+(\vec{k}-\vec{k}')^2}$$

$$d^3k' = dk' k'^2 d\hat{k}' = dk' k'^2 d\phi \, d\Theta \sin\Theta$$

$$\rho = d\hat{k}' dk' k'^2 \delta(\underbrace{k_0+M-k_0'-\tilde{P}_0'}_{f(k')})$$

$$f(k') = k_0+M-\sqrt{m^2+k'^2}-\sqrt{M^2+k^2+k'^2-2k\cos\Theta k'}$$

$$f'(k') = -\frac{2k'}{2k_0'} - \frac{2k'-2k\cos\Theta}{2P_0'} =$$

$$|f'(k')| = \frac{k'}{k_0'} + \frac{k'-k\cos\Theta}{P_0'}$$

$$\rho = \frac{d\hat{k}' \, k'^2}{\left| \frac{k'}{k_0'} + \frac{k' - k\cos\Theta}{p_0'} \right|}$$

Czynnik przestrzeni fazowej możemy też zapisać w innej postaci

$$k' = \sqrt{E'^2 - m^2} \quad | \; E' = k_0'$$

$$dk' = \frac{2E'}{2k'} dE' = \frac{E'}{k'} dE'$$

$$dk' \, k'^2 = E' k' dE'$$

$$\rho = d\hat{k}' dE' E' k' \, \delta\big(\underbrace{k_0 + M - E' - \sqrt{M^2 + k^2 + E'^2 - m^2 - 2k\cos\Theta \sqrt{E'^2 - m^2}}}_{g(E')}\big)$$

$$g'(E') = -1 - \frac{2E' - 2k\cos\Theta \frac{2E'}{2k'}}{2p_0'}$$

$$|g'(E')| = \left| 1 + \frac{E' - \frac{E' k\cos\Theta}{k'}}{p_0'} \right|$$

$$= \frac{p_0' + E' - \frac{E' k \cos\Theta}{k'}}{p_0'}$$

$$|g'(E')| = \frac{(k_0+M)k' - E'k\cos\Theta}{P_0'k'}$$

$$\rho = \frac{E'k'P_0'k'\,d\hat{k}'}{(k_0+M)k' - E'k\cos\Theta} =$$

$$= \frac{E'P_0'k'\,d\hat{k}'}{(k_0+M) - \frac{kE'}{k'}\cos\Theta}$$

p. 1

$$d\sigma^{LAB} = \frac{\cancel{k_0}}{k}\frac{1}{2\cancel{k_0}}\frac{1}{2M}\frac{1}{2k_0'\,2p_0'}\frac{(2\pi)^4}{(2\pi)^6}\int$$

$$|\mathcal{M}|^2\,\delta$$

$$d\sigma^{LAB} = \frac{1}{16}\frac{1}{(2\pi)^2}\frac{1}{Mk\,k_0'p_0'}\int|\mathcal{M}|^2$$

$$\frac{k_0'p_0'k'}{(k_0+M)-\frac{kk_0'}{k'}\cos\Theta}$$

$$d\sigma^{LAB} = \frac{1}{16}\frac{1}{(2\pi)^2}\frac{k'}{Mk}\int\frac{d\hat{k}'}{(k_0+M)-\frac{kk_0'}{k'}\cos\Theta}$$

$$\sum_{\substack{s,s'\\t,t'}}\left|\bar{u}(k's')\gamma_\beta u(ks)\,\bar{u}(p't')\gamma^\beta u(p,t)\right|^2$$

$$\frac{(e^2)^2}{(q^2)^2}$$

$$(e^2)^2 = (4\pi\alpha)^2,\quad \alpha\simeq\frac{1}{137},\quad \int=1$$

$$(q^2)^2 = \left((k_0-k_0')^2-(\vec{k}-\vec{k}')^2\right)^2$$

Granica niskoenergetyczna

$k/m \to 0 \quad , \quad k_0/M \to 0$

$k_0' = k_0 \, , \, k' = k$ ("odbicie od ściany")

Strona (14)

$$A \approx 16 M^2 k_0^2 + 8 m^2 M^2 - 8 M^2 (k \cdot k')$$

$$\frac{d\sigma^{lab}}{d\hat{k}'} \approx \frac{1}{16} \frac{1}{(2\pi)^2} \underbrace{\frac{1}{M} \frac{1}{(k_0 + M) - k_0' \cos\Theta} M^2}_{\simeq 1}$$

$$\cdot \frac{2}{2} \cdot \frac{(4\pi\alpha)^2}{(\vec{q}^2)^2} 8 \left(2 k_0^2 - (k \cdot k') + m^2\right)$$

$$\approx \frac{2\alpha^2}{(\vec{q}^2)^2} \left(2 k_0^2 - (k \cdot k') + m^2\right)$$

$$\underbrace{\frac{1}{16} \frac{1}{(2\pi)^2} \frac{(4\pi)^2}{2} \cdot 8}$$

$$\underbrace{\frac{1}{16} \cdot 4 \cdot \frac{1}{2} \cdot 8}_{1}$$

Granica ultrarelatywistyczna

$k \gg m, \ k' \gg m, \ k_0 = k, \ k_0' = k'$

p. 18 & 14'

$$\frac{d\sigma^{lab}}{d\Omega'} \approx \frac{1}{16}\frac{1}{(2\pi)^2}\frac{k'}{Mk}\frac{1}{(k_0+M)-k\cos\Theta}$$

$$\frac{(4\pi\alpha)^2}{(q^2)^2} 8 \Big(-(k\cdot k')\left(M^2+M(k_0'-k_0)\right) \checkmark$$
$$+ 2M^2 k_0 k_0' \checkmark$$
$$+ 2m^2 M(k_0'-k_0)$$
$$+ m^2 M^2 \Big)$$

$$\approx \frac{1}{16}\frac{1}{(2\pi)^2}\frac{k'}{kM^2}\frac{1}{1+\frac{k_0}{M}-\frac{k_0}{M}\cos\Theta}$$

$$\frac{(4\pi\alpha)^2}{(q^2)^2} 8 \Big(2M^2 k_0 k_0' + \frac{q^2}{2}\left(M^2+M(k_0'-k_0)\right)$$
$$+ 2m^2 M(k_0'-k_0) \Big)$$

$$= \frac{1}{16}\frac{1}{(2\pi)^2}\frac{k'}{kM^2}\frac{1}{1+\frac{2k_0}{M}\sin^2\frac{\Theta}{2}}$$

$$\frac{(4\pi\alpha)^2}{(q^2)^2} 8\cdot 2M^2 k_0 k_0' \left(1+\frac{q^2\left(M^2+M(k_0'-k_0)\right)}{4M^2k_0k_0'} + \frac{2m^2M(k_0'-k_0)}{2M^2k_0k_0'}\right)$$

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$$= \frac{1}{(2\pi)^2} \frac{k'}{k} \frac{1}{1+\frac{2k_0}{M}\sin^2\frac{\Theta}{2}}$$

$$\frac{(4\pi\alpha)^2}{(q^2)^2} k_0 k_0' \left(1 + \frac{q^2}{4k_0k_0'}\left(1+\frac{k_0'-k_0}{M}\right) + \underbrace{\frac{m^2}{k_0k_0'}\frac{(k_0'-k_0)}{M}}_{\approx 0}\right)$$

$$q^2 \simeq -4k_0k_0'\sin^2\frac{\Theta}{2}$$

$k+P-k' = p'$ (rozpraszanie elastyczne)

$$M^2 = p'^2 = (k+p-k')^2 =$$

$$= k^2+p^2+k'^2+2k\cdot p-2k\cdot k'-2p\cdot k'$$

$$= \underline{m^2}+M^2+\underline{m^2}+\underline{2k_0M}-2k\cdot k'-\underline{2k_0'M}$$

$$0 = 2m^2+2M(k_0-k_0')-2k\cdot k'$$

$$2M(k_0'-k_0) \approx -2k\cdot k' =$$

$$= -2k_0k_0'+2kk'\cos\Theta$$

$$\simeq -2kk'(1-\cos\Theta) = -4kk'\sin^2\frac{\Theta}{2}$$

$$\approx q^2$$

$$\Rightarrow \frac{k_0'-k_0}{M} \approx \frac{q^2}{2M^2} \quad \text{oraz} \quad \frac{q^2}{4kk'} \simeq -\sin^2\frac{\Theta}{2}$$

$$\frac{d\sigma^{lab}}{d\vec{k}'} \approx \frac{1}{(2\pi)^2} \frac{k'^2}{1+\frac{2k_0}{M}\sin^2\frac{\theta}{2}} \frac{(4\pi\alpha)^2}{16k^2k'^2\sin^4\frac{\theta}{2}} \left(1+\frac{q^2}{4k_0k_0'} + \frac{q^2}{4k_0k_0'}\frac{q^2}{2M^2}\right)$$

$$= \frac{\alpha^2}{4k^2\sin^4\frac{\theta}{2}} \cdot \frac{\left(1-\sin^2\frac{\theta}{2} - \frac{q^2}{2M^2}\sin^2\frac{\theta}{2}\right)}{1+\frac{2k_0}{M}\sin^2\frac{\theta}{2}}$$

$$= \frac{\alpha^2}{4k^2\sin^4\frac{\theta}{2}} \frac{\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\sin^2\frac{\theta}{2}}{1+\frac{2k_0}{M}\sin^2\frac{\theta}{2}}$$