

Ilustracja twierdzenia Cayleya

Szukamy podgrupy S_4 izomorficznej z grupą $D_2 = \{e, b, c, bc\}$, $b^2 = c^2 = (bc)^2 = e$

Tabela mnożenia grupowego dla D_2

	e	b	c	bc
e	e	b	c	bc
b	b	e	bc	c
c	c	bc	e	b
bc	bc	c	b	e

$$cb = cbe = \underbrace{cb}_{e}bc = \underbrace{cc}_{e}bc = bc$$

$$\varphi(e) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} = () = E$$

$$\varphi(b) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix} = (12)(34) = B$$

$$\varphi(c) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix} = (13)(24) = C$$

$$\varphi(bc) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix} = (14)(23) = \overline{BC}$$

$$BB = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} = E$$

$$BC = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix} = \overline{BC}$$

$$B\overline{BC} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix} = C$$

$$CB = \begin{pmatrix} 1234 \\ 3412 \end{pmatrix} \begin{pmatrix} 1234 \\ 2143 \end{pmatrix} = \begin{pmatrix} 1234 \\ 4321 \end{pmatrix} = \overline{BC}$$

$$CC = \begin{pmatrix} 1234 \\ 3412 \end{pmatrix} \begin{pmatrix} 1234 \\ 3412 \end{pmatrix} = \begin{pmatrix} 1234 \\ 1234 \end{pmatrix} = E$$

$$C\overline{BC} = \begin{pmatrix} 1234 \\ 3412 \end{pmatrix} \begin{pmatrix} 1234 \\ 4321 \end{pmatrix} = \begin{pmatrix} 1234 \\ 2143 \end{pmatrix} = B$$

$$\overline{BC}C = \begin{pmatrix} 1234 \\ 4321 \end{pmatrix} \begin{pmatrix} 1234 \\ 3412 \end{pmatrix} = \begin{pmatrix} 1234 \\ 2143 \end{pmatrix} = B$$

$$\overline{BC}B = \begin{pmatrix} 1234 \\ 4321 \end{pmatrix} \begin{pmatrix} 1234 \\ 2143 \end{pmatrix} = \begin{pmatrix} 1234 \\ 3412 \end{pmatrix} = C$$

$$\overline{BC}\overline{BC} = \begin{pmatrix} 1234 \\ 4321 \end{pmatrix} \begin{pmatrix} 1234 \\ 4321 \end{pmatrix} = \begin{pmatrix} 1234 \\ 1234 \end{pmatrix} = E$$

$$H \subset S_4, H \cong D_2$$

$$H = \{(), (12)(34), (13)(24), (14)(23)\}$$

$$D_2 = \{e, b, c, bc\}$$

Szukamy podgrupy S_4
izomorficznej z grupą C_4

$$C_4 = \{e, c, c^2, c^3\}, c^4 = e$$

$$P(e) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} = () = E$$

$$P(c) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix} = (1234)$$

$$P(c^2) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix} = (13)(24)$$

$$P(c^3) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix} = (1432)$$

$$P(c) P(c) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$$

$$P(c) P(c) P(c) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$$

$$P(c) P(c) P(c) P(c) =$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} = E$$

$$H' \subset S_4, H' \cong C_4$$

$$H' = \{ (), (1234), (13)(24), (1432) \}$$

$$C_4 = \{ e, c, c^2, c^3 \}$$