Defect-assisted hard x-ray microscopy with capillary optics

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Polycapillary x-ray focusing devices are built from hundreds of thousands of bent glass microcapillaries that are stacked into hexagonal arrays. We show that intrinsic point defects of the optics (missing or larger capillaries) lead to the formation of multiple x-ray images of an object, which was positioned in the focal plane. These images can be recorded in parallel, and provide spatial resolution that is limited by the defect size and not by the focal spot size. In a proof-of-principle experiment, we demonstrate sub-micron resolution that has never been achieved with polycapillary focusing optics. Tailored optics with a controlled distribution of “defects” could be used for multimodal nanoscale x-ray imaging with laboratory setups.

For x-rays, the refractive index is slightly smaller than unity [1, 2] and hollow microcapillaries [3–9] or nanochannels [10, 11] are capable to guide x-rays by successive total external reflections. Polycapillary x-ray focusing elements are built from hundreds of thousands of such bent glass microcapillaries that are stacked into hexagonal arrays [12]. They are fabricated using a repeated stack-and-draw process from fibre optics technology [13] and enable to concentrate or focus x-rays from laboratory or synchrotron sources into intense spots. The focal spot of a polycapillary device is formed by an incoherent overlap of divergent beams from all capillaries and has a lateral diameter of \( \Delta x \approx 2f\theta_c [12] \). The focal length \( f \), with usually few millimeters to several tens of millimeters, is determined by the bending radius of capillaries. For borosilicate glass, the critical angle for the total external reflection can be approximated as \( \theta_c [\text{mrad}] = 30/E [\text{keV}] \), where \( E \) is the energy of the x-ray beam. Therefore, in the hard x-ray range, the focal spot typically has a diameter between 10 and 100 \( \mu m \) and polycapillary “lenses” are most frequently applied in scanning micro x-ray fluorescence (\( \mu XRF \)) [14–19]. Polycapillary devices are achromatic and have very large angular apertures (up to 20°). They are especially suited for collecting polychromatic radiation from a large solid angle in x-ray tube-based applications. Attached to x-ray tube, they provide focal spots with sizes that are comparable with that of single-bounce capillaries but which are much more intense [20]. However, microscopy with focusing polycapillary optics has never reached sub-micron [6] or nano-resolution (<100 nm) [8], which was demonstrated in synchrotron beam experiments with single tapered capillaries and which is limited by the diameter of the capillary channel.

While a polycapillary optics is not classical imaging optics [2, 9, 21], the spatial resolution in both projection and scanning imaging with polycapillaries is limited by the focal spot size \( \Delta x [22, 23] \), similarly as for x-ray imaging elements i.e. for compound refractive lenses [24], focusing mirrors [25] or zone plates [26]. Only recently it was demonstrated that it is possible to resolve details of an object placed inside the focal spot of a polycapillary optics [22]. Details (at a resolution of approx. 10 \( \mu m \)) could be decoded from a distorted image of the periodic superstructure of capillary bundles which was treated as a coding aperture [27–29]. It was also shown that x-ray coded aperture imaging with polycapillary optics provides depth resolution without sample or source rotation, in a way similar to classical tomography or laminography [30]. However, due to the fundamental constrains imposed by the Shannon sampling theorem [31, 32], coded aperture approach cannot be extended for the analysis of the periodic structure of individual capillaries. In large periodic capillary arrays, structural defects are inherently present, but in all previous reports defects in capillary arrays were either neglected or discussed in the context of a decreased transmission and a deteriorated performance [15, 33–35].

In this work, we demonstrate that intrinsic point defects (missing or larger capillaries), by breaking the periodicity of capillary arrays, directly lead to the formation of multiple x-ray images of an object placed inside the focal spot of polycapillary optics. Such multiple images can be analyzed in parallel which enhances the imaging performance and provides a spatial resolution that is limited by the defect size and not by the focal spot size of the optics. In a proof-of-principle experiment, we obtained sub-micron resolution (~0.6 \( \mu m \)) that has been not reported with polycapillary focusing optics until now. The presented defect-assisted concept of microscopy combines the high spatial resolution provided by single capillaries with the high x-ray flux delivered by polycapillary structures and thereby is promising in the context of multimodal nanoscale imaging with polychromatic low brilliance laboratory x-ray sources.

The idea of defect-assisted x-ray microscopy is presented in Fig. 1 If an object is placed in the focal plane of the optics, the intensity \( I(r) \) of x-rays recorded with a 2D detector can be approximated by a convolution [35]:

\[
I(r) = \frac{1}{S_0M^2} [T_M(r)F_M(r)] \ast S_M(r),
\]

where \( T_M \) is the transmission of the object, magnified by a factor of \( 1 - M \) (see Supplemental Material [36]), \( F_M \)
In order to demonstrate defect-assisted x-ray microscopy, we performed an experiment with polychromatic radiation from a tungsten anode x-ray tube (XTG5011 Apogee, Oxford Instruments) with 40 μm spot operated at 50 kV and 1 mA. The object, letter “E” (6.2 μm × 4.5 μm) was milled with a focused ion beam (Dual Beam SEM/FIB Quanta 3D FEG, FEI) in an Au foil and was placed inside the focal spot of the optics. The optics (micro-lens for x-ray fluorescence spectroscopy, HFG) had a focal length $f \approx 2.5$ mm, exit aperture of 1.1 mm and produced a 11.8 μm focal spot [c.f. Fig. 2(b)]. The optics consisted of approx. $3.3 \times 10^5$ capillaries arranged in 463 bundles. Transmitted x-rays were recorded using a photon counting detector (Timepix, WidePix) having $256 \times 256$ pixels with pitch 55 μm that was placed at a distance $D = 447$ mm from the optics. This detector covered only a small fraction (approx. 1%) of the radiation cone generated by the optics. The optics and the object were placed on piezo XYZ stages (MX35 and MS30, Mechnics). The effective energy of x-rays was around 9 keV. For details of the experimental setup see Supplemental Material [36].

First, for a visualization and location of defects, we placed a 0.8 μm diameter pinhole inside the focal spot. The pinhole was milled in the same foil as the object. A pinhole approximates a point-like object with a transmission $T_M(r) = \delta(r)$. For such an object, Eq. (1) becomes $I(r) \propto F(0)S_M(r)$ and the x-ray image [Fig. 2(a)] shows the magnified x-ray distribution at the exit surface of the optics. All x-ray images are presented as $I/I_0 \times \overline{T_0}$, where $I$ and $I_0$ were recorded with and without the object, respectively, and $\overline{T_0}$ is the mean value of the image recorded without the object. With this data presentation, it was possible to show both the contrast and the total photon counting rates. For long exposures, final images were combined from multiple 30 s frames that were corrected for drifts by using the periodic pattern of the capillary structure as the position indicator. In Fig. 2(a), we could observe individual capillaries with a spacing of $a = 1.27 \pm 0.05 \mu m$ and with channel diameters at the level of 1 micron. Most characteristic structural defects were marked with labels p1-p4 and are shown at higher magnification in Fig. 2(b). Most frequently, defects are located at the vertices of the hexagons at the boundaries between bundles. However, they are also present in the central parts of the bundles. For example, in the areas marked with labels p3 and p4 we could observe isolated defects corresponding to broken capillaries and to capillaries with a higher transmission (most probably having a slightly larger diameter). The low-frequency modulation of the capillary image is due to a small angular misalignment of capillaries.

After placing the object inside the focus of the optics [Fig. 2(d)], we observed the formation of distinct images of the object at locations corresponding to the positions of defects. The contrast of particular images (e.g. dark

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**FIG. 1.** Principle of defect-assisted x-ray microscopy. (a) Experimental setup. The object is placed inside the focal spot of a polycapillary focusing optics which has naturally defected capillaries. Transmitted x-rays are detected using a position sensitive detector. (b) Focal spot of the optics measured using a pinhole scan. Inset shows SEM image of the object at the same scale. (c) Simulation of image formation. The image in the detector can be calculated as a convolution of the magnified object’s transmission and the distribution of x-rays from capillaries. For a perfect periodic arrangement of capillaries, information about the object is smeared. Defects (missing or larger capillaries marked with dashed circles) break the periodicity and lead to the formation of multiple x-ray images of the object.

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describes the Gaussian-like spatial distribution of radiation in the focal spot [c.f. Fig. 1(b)], at a magnification $M$, and $S_M$ represents the spatial distribution of radiation at the exit plane of the optics, at the same magnification. $S_M$ is determined by the arrangement of the capillaries and their transmission properties. $M$ is defined as $M = (f - D)/f$, where $D$ is the detector-to-optics distance and $S_0$ is the total number of photons in the focal spot.

The focal spot of polycapillary optics is always much greater than the spatial period of the capillaries. Therefore, for a periodic $S_M$, the convolution operation smears out the information about the object [see Fig. 2(c)]. The resulting image has the same periodicity as the capillary structure. It encodes some information about the object, but only at a set of discrete spatial frequencies that are too sparsely spaced for a well-posed object reconstruction [36]. Point defects (missing, broken or slightly larger capillaries) change the situation, as shown by the simulation in Fig. 2(c). Such defects give rise to the formation of multiple images that are superimposed on the periodic pattern of ordered capillaries. Therefore, defect-assisted microscopy, in some ways, is similar to incoherent multiple-pinhole [27, 42, 43] or coherent multiple-reference x-ray imaging [29, 44].
FIG. 2. Demonstration of defect-assisted x-ray microscopy. (a) Visualization of defects. X-ray image of a small part of the optics (~1% of the total aperture) obtained by placing a 0.8 µm diameter pinhole inside the focal spot of the optics. (b) Zoom of typical structural defects. (c) Fourier transform of the image from (a). (d) Formation of x-ray images due to defects. The object (letter “E”) is placed inside the focal spot. (e) Image from (d) after removal of the periodic component. (f) Fourier transform of image (d). Insets in left-bottom corners: SEM images of the pinhole and the object. Color bars: total number of photons recorded in each detector pixel (gray) or logarithm of Fourier transform in arbitrary units (false color scale).

For p3 and white for p4) depends on the type of the defect. For defect complexes (e.g. p1 and p2) we could observe an overlap of multiple images. The visibility of images was enhanced in Fig. 2(e) by suppression of the periodic component using a Fourier filter.

For a parallel reconstruction [Fig. 3(a)] of the object’s transmission from multiple images, we used the coded-aperture approach [22]. The object’s transmission $T_M$ was decoded by applying a Wiener deconvolution [45] to the image $I$ from Fig. 2(e) and using the image of capillary pattern from Fig. 2(a) as the coded aperture $S_M$. The signal-to-noise (SNR) parameter of the Wiener filter was frequency independent. It’s optimal value was determined by maximizing SNR in the reconstruction (ratio of the signal power inside and outside the object area).

In order to determine the spatial resolution we have calculated the system point spread function (SPSF). It is a quantitative measure of the resolution in the coded-aperture approach [28]. The SPSF is defined as the reconstructed image of a point object (in our case approximated by the pinhole). It shown in Figs. 3(c) and 3(d). Analysis of the profiles of the SPSF indicates that the spatial resolution is at the level of 600 nm (the Nyquist frequency in Figs. 2(c) and 2(f) corresponds to a half-period spatial resolution of 310 nm). The small side-lobes of the SPSF curve result from the non-perfect suppression of the periodic signal.

The image presented in Fig. 2(d) was recorded in a 200 min exposure. Such a long exposure was used to obtain clear images formed by each individual defect and to analyse the reconstruction resolution for different types of defect complexes [36]. However, coded aperture imag-
FIG. 3. Comparison of defect-assisted microscopy with standard x-ray projection imaging with the focal spot acting as a secondary source. (a) Object reconstructed using the coded aperture approach from multiple defect-assisted images (dark - low transmission, bright high transmission). (b) X-ray image of the object positioned out-of-focal plane at \( f + 2.5 \) mm. The resolution is limited by the penumbral blur resulting from the large spot size. Color bar: photons/s/pixel. (c) System point spread function (SPSF) of defect-assisted imaging. (d) Profiles of the SPSF interpolated from (c) along marked lines. Arrowed lines: FWHM (0.50 ± 0.15 \( \mu m \)) of the red dashed curve. Dashed vertical line: position of the first minimum (0.59 ± 0.15 \( \mu m \)) of the blue solid curve. Insets: SEM images of the object and of the 0.8 \( \mu m \) pinhole.

FIG. 4. Efficiency of defect-assisted x-ray microscopy. (a) X-ray images of the object for 10 and 100 times shorter exposure times as compared to data from Fig. 2(e). Left: \( t = 20 \) min. Right: \( t = 2 \) min. Only small fragments corresponding to the area marked with a white square in Fig. 2(e) are shown. (b) Reconstructions of the object from multiple defect-assisted images for the corresponding exposure times.

Concluding, defect-assisted microscopy achieved sub-micron resolution, which corresponds to an order of magnitude improvement as compared to the focal-spot limited resolution of polycapillary optics. It combines the high spatial resolution given by single capillaries with the high flux provided by polycapillary structures. Therefore, tailored devices could be used for multi-modal i.e absorption, in-line phase contrast \([46-49]\) and \( \mu \)XRF \([17, 18]\) imaging of real samples with incoherent laboratory x-ray sources. Efficient megapixel detectors and optics “doped” with defects (solid fibers or sparse capillaries), manufactured using technology known from photonic crystal fibres \([50, 51]\), would provide a massive improvement in signal-to-noise ratio (see Supplemental Material \([36]\)). The spatial resolution is limited by the size of defect, hence it is at the level of capillary channel diameter. For a single tapered capillary, the downsizing of the channel diameter to less than 100 nm is possible \([8]\). It was also demonstrated that straight nanocapillary arrays are capable to transport x-rays \([52]\) and very recently it was shown that nanochannels bend to several tens of degrees can be still used to guide x-ray photons \([11]\). Bent nanocapillary arrays could be very fragile and defected. However, in this work, we demonstrated that the defects are very useful.

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IMAGE FORMATION

In a recent paper we introduced a model of image formation in x-ray coded aperture microscopy with polycapillary optics that enabled a direct incorporation of optics defects. However, in Ref. [S1], as in all previous reports, defects were discussed only in the context of image quality degradation. Nevertheless, the introduced model can be directly used for description and simulation of defect-assisted image formation in capillary arrays reported in this Letter. Here we briefly summarize the derivation of Eq. (1) from the Letter.

The geometry used in the calculation is shown in Fig. S1. The exit surface of the polycapillary optics is treated as an extended and micro-structured x-ray source. Emittance is determined by the microstructure of the optics as well as the transmission properties of individual capillaries. It is assumed that radiation from individual capillaries is emitted in a shallow cone directed towards the focus that has an opening angle governed by the critical angle for the total external reflection.

As shown in Ref. [S1], the intensity recorded by a detector placed at a distance $D$ from the exit surface optics of the polycapillary optics, and with an object located at $z = d$ can be written as:

$$I(r) = \frac{1}{D^2} \int T \left( \frac{r - r_s}{D} d + r_s \right) G(r - Mr_s) S(r_s) dr_s,$$  \hspace{1cm} (S1)

where $T$ is the transmission of the object, $r_s$ and $r$ are the lateral positions at $z = 0$ and $z = D$ planes, respectively. $S$ describes the microstructure of the optics as well as the transmission...
properties of the optics. The magnification factor is defined as

\[ M = \frac{(f - D)}{f}, \tag{S2} \]

where \( f \) is the focal length. \( G \) describes the angular properties of radiation emitted from the optics. Radiation from each capillary is emitted towards the focus in a shallow cone, hence \( G \) can be written as:

\[ G(r) = \exp \left[ -0.5 \frac{|r|^2}{(D\theta_c)^2} \right] / (2\pi\theta_c^2), \]

where \( \theta_c \) is the critical angle for the total external reflection.

Equation (1) from the Letter can be obtained from Eq. (S1) by putting \( d = f \) and introducing the following functions:

\[ T_M(r) = T \left( \frac{r}{1 - M} \right), \tag{S3} \]

\[ S_M(r) = S \left( \frac{r}{M} \right), \tag{S4} \]

which describe the magnified images of the object and the capillary structure, respectively. \( S_0 \) is equal to the total flux generated by the optics \( S_0 = \int S(r_s)dr_s \).

For the purpose of this Letter, it is useful to introduce a new quantity \( F \), that describes the distribution of the radiation at the focal plane of the optics and that can be directly measured in the experiment [c.f Fig. 1(b)]. The intensity distribution at the focal plane \( F \) can be related to the angular distribution of radiation emitted from capillaries \( G \) by:

\[ G(r) = \frac{D^2}{S_0} F \left( \frac{r}{M} \right) = \frac{D^2}{S_0} F_M(r). \tag{S5} \]

The background intensity without the object can be expressed as:

\[ I_0(r) = \frac{1}{S_0 M^2} F_M(r) * S_M(r) \tag{S6} \]

and corresponds to a magnified but a very blurred image of the capillary structure \( S \).

For an object displaced from the focal plane \( (d \gg f) \), the image formation is equivalent to a point projection geometry with the focal spot of the optics acting as a secondary source and resolution limited by the penumbra blur resulting from the finite size of the focal spot. In practice, as indicated by the experimental data, projection geometry for our setup is already observed for \( d \gtrsim 2f \).
OPTICS

We used a commercially available polycapillary optics for micro x-ray fluorescence spectroscopy. The optics consisted of approx. $N = 3.3 \times 10^5$ capillaries (463 bundles, each having on average 721 capillaries) and had a focal length $f = 2.5 \pm 0.1$ mm. The focal spot shape $F(r)$ had an approximate Gaussian shape with FWHM of $\Delta x = 11.8 \pm 0.8$ $\mu$m (see Fig. 1(b) of the Letter). The total number of photons per second in the spot was $2.9 \times 10^8$ ph/s/mA (as measured with a 300 $\mu$m thick Si hybrid pixel detector, i.e. without correction for the detection efficiency).

The mean energy of the x-ray beam as well as energetic and spatial dependencies of optics transmission $\tau(E, r_s) = I_{\text{output}}(E, r_s)/I_{\text{input}}(E)$ were measured directly using the hybrid pixel detector. Due to the charge sharing effects, which result in a low-energy tail of x-ray spectra, the presented energy-dependent data (contrary to energy-integrated data) have large uncertainties (especially in the 5-8 keV range) and are only approximate. For the determination of the number of photons at the output surface $I_{\text{output}}(E, r_s)$, the detector was placed at a distance $D = 46.6$ mm. At such a short distance the detector covered the whole radiation cone generated by the optics. For the determination of $I_{\text{input}}(E)$, the optics was removed, the detector was moved to $D = 446.6$ mm (in order to keep the total flux at a similar level) and the number of photons at the input surface of the optics was calculated from the knowledge of the solid angles. Results are summarized in Fig. S2 and Fig. S3. Relative uncertainties in the measured quantities result from uncertainties of the nominal geometrical parameters of the optics and are at the level of 10%.
FIG. S2. Measured energy-integrated transmission of the optics. (a) Spatial dependence of optics transmission. (b) Radial average of the transmission. Horizontal dashed line marks the mean transmission of the optics $\tau_{\text{mean}} = 0.016$. Maximum transmission at the center of the optics is $\tau_{\text{center}} = 0.089$. The white rectangle in (a) corresponds to the solid angle covered by the detector during x-ray imaging. During the defect-assisted imaging, the detector was intentionally shifted from the optical axis and the image was formed by capillaries with a high bending angle of approx. 9°.

FIG. S3. Measured position-integrated transmission of the optics. (a) Comparison of maximum-normalized energy spectra with (red circles) and without optics (blue squares). The spectra were measured with the FitPIX detector. The vertical red line shows mean energy without optics (9.9 keV), the vertical blue line shows the mean energy with optics (9.1 keV). Both values can be underestimated by approximately 1 keV due to the charge sharing effect. The spectrum at the bottom (black thin line) was recorded using a silicon drift detector at an arbitrary position and is shown to explain the shape of the FitPIX spectra. (b) Transmission of the optics. The horizontal dashed line marks the mean transmission of the optics.
FIG. S4. Object used in the experiment. Letter "E" (6.2 µm × 4.5 µm, width of the horizontal lines of ~0.9 µm) and pinholes with various sizes were milled using FIB at a rim of a 4 µm thick electron microscopy Au grid. The visible grid has a pitch of 12.5 µm (mesh 2000) and was used for the determination of the focal plane position with an accuracy of ±10 µm. (a) SEM image. Scale bar in the inset: 5 µm. (b) X-ray projection image measured with the object placed out-of-focus. The rectangle marks the area shown in Fig. 3(b) of the Letter. Note that the resolution of the standard x-ray projection is limited by the size of the focal spot of the optics. Nevertheless, such projection images are very useful for moving the object into the focal spot for the high-resolution defect-assisted imaging. Color bars: total number of photons per pixel.
RECONSTRUCTION

FIG. S5. Reconstruction of the object from periodic and diffuse components of the data. Object reconstruction from the periodic component alone was impossible. The signal is massively undersampled. From left to right: Fourier transform (FT) of data from Fig. 2a, FT of data from Fig. 2d, object reconstruction, SPSF, and profiles of SPSF. Upper row show data without filtering. Middle row show data after suppression of the diffuse component of the FT. Lower row show data after suppression of the periodic component of the FT.
FIG. S6. Reconstruction of the object from small regions around defect complexes (p1,p2) or isolated (p3,p4) point defects. Rows correspond to regions p1-p4 from Fig. 2 of the Letter:
p1 - example of a defect complex - two missing capillaries ("bi-vacancy") and a few minor defects ,
p2 - example of a defect complex - missing capillary ("vacancy") and a few minor defects
p3 - example of an isolated defect - capillary with a decreased transmission,
p4 - example of an isolated defect - capillary with an increased transmission.
(a) Images of the defects (multiplied by a Gaussian window that limits the effective data range). (b) Defect-assisted images of the object (after a Fourier filter). (c) Object reconstructions from limited data range. (c) Profiles of the SPSF. The numbers in each SPSF plot correspond to FWHM of the red dashed curve and to the position of the first minimum of the blue solid line. The resolution is slightly deteriorated compared to the reconstruction from full x-ray pattern due to noise that influences the high frequency components of the data. The reconstruction for isolated defects (p3 and p4) is superfluous (such defects directly form images) and it is shown only for a direct comparison of the resolution and to show how the signal from such defects contributes to the total reconstruction from Fig. 3(a) of the Letter.
EFFICIENCY

It is useful to define the gain $G_0$ in the intensity at the focal spot provided by polycapillary optics relative to a single tapered capillary located at the optical axis:\(^{S4}\):

$$G_0 = N\eta,$$  \hspace{1cm} (S7)

where

$$\eta \approx 0.9 \frac{\tau_{\text{mean}}}{\tau_{\text{center}}}$$  \hspace{1cm} (S8)

and $\tau_{\text{mean}}$ and $\tau_{\text{center}}$ are the mean and maximum transmission of the polycapillary optics, respectively. The numerical pre-factor is the hexagonal filling ratio of capillaries. In our case $G_0 \approx 5.5 \times 10^4$. Such a high value of $G_0$ is a hallmark of polycapillary optics and provides the basis of efficient micro x-ray fluorescence ($\mu$XRF) experiments even with laboratory sources.

Simultaneously, defect-assisted microscopy achieved a resolution $\sim \delta x / 2$, where $\delta x$ is the diameter of capillary channels. Therefore, the gain in the resolution (relative to the focal-spot limited resolution) is:

$$G_{\text{res}} \approx \Delta x / \delta x \approx 2 / f\phi_c \delta x$$  \hspace{1cm} (S9)

where $\phi_c$ is the critical angle for the mean beam energy. In the experiment we obtained $G_{\text{res}} \approx 10$ ($\Delta x / 2 \approx 5.9 \ \mu m$, $\delta x / 2 \approx 0.59 \ \mu m$).

It is also worthy to provide an expression for the signal-to-nose ratio (SNR). Since the background related noise is dominant, for simplicity we do not take into account the decoding noise, which depends on the applied decoding scheme and on details of the coding pattern.

For a single capillary, SNR is:\(^{S5}\):

$$\text{SNR}_1 = \frac{\Delta TI_1 A}{\sqrt{TI_1 A}} = C_1 \sqrt{TI_1 A}$$  \hspace{1cm} (S10)

where $C_1 = \Delta T / T$ is the contrast, $I_1$ describes the number of photons emitted by a single capillary, $T$ is the mean object transmission, $\Delta T$ are the variation of the object’s transmission and $A$ is the area of the pixel. Here, $T \approx 0.6$ and for weakly absorbing samples $T \rightarrow 1$. The value of $C_1$ for our object can be extracted from Fig. S4b, where the maximum measured contrast is $C_1 \approx 1.35$.

In defect-assisted imaging, the signal is formed by $N_d$ defects and multiplied. For the moment assume that $N_d$ describes only intrinsic broken or missing capillaries or intentionally ”doped” solid fibers. However, since the signal is superimposed on a background coming from capillaries in the local neighborhood of the defect, the contrast will be simultaneously decreased. As seen from Eq. S6, the background signal originates only from the area around the defect that is determined by the focal spot size. Hence, the number of capillaries $N_B$ contributing to the background can be estimated as the ratio of the effective area of the focal spot $\int F(r)dr$ and the area occupied by a single capillary:

$$N_B \approx 0.9 \frac{2\pi(f\phi_c)^2}{\pi(a/2)^2} \approx 0.9 \frac{2\pi(\Delta x / 2.35)^2}{\pi(a/2)^2}$$  \hspace{1cm} (S11)

where $a$ is the lattice constant of the capillary structure. Consequently, contrast is diminished and becomes:

$$C_{\text{poly}} = \frac{C_1}{N_B(1 - n_d)},$$  \hspace{1cm} (S12)
where $n_d = N_d/N$. Here, $n_d \approx 0.01$ and the number of capillaries effectively contributing to the background is $N_B \approx 112$. Therefore, the value of contrast becomes $C_{\text{poly}} \approx 0.012$. This value excellently agrees with the contrast observed in Fig. 2e of the letter which is at the level of $4 \times 10^3/3.3 \times 10^5 \approx 0.012$.

Using similar arguments as in Ref. [S6], the signal-to-noise ratio for defect-assisted microscopy with polycapillary optics can be written as:

$$\text{SNR}_{\text{poly}} = C_{\text{poly}} \sqrt{T I_1 G_0 n_d (1 - n_d) N_B A \Omega},$$

(S13)

where $\Omega$ is the fraction of the angular aperture of the optics that is covered by the detector. Finally, the gain in SNR can be written as:

$$G_{\text{SNR}} = \frac{\text{SNR}_{\text{poly}}}{\text{SNR}_1} = \sqrt{\frac{G_0 n_d \Omega}{N_B (1 - n_d)}}$$

(S14)

In our proof-of-principle experiment we used a small detector ($\Omega \approx 0.01$) and random sparse defects ($n_d \approx 0.01$) and therefore a small value of $G_{\text{SNR}} \approx 0.22$ was achieved. Despite this, the intensity provided by the optics was sufficient to obtain sub-micron resolution with polycapillary optics for the first time. However, the optics used in the experiment (designed for $\mu$XRF) is capable to provide $G_{\text{SNR}} > 2$, with a larger detector ($\Omega \approx 1$). An introduction of 20% random defects (solid fibers) into a tailored device would provide a gain of $G_{\text{SNR}} > 10$ with only a 20% loss in the total flux in the focal spot. This denotes 100 times shorter experiments as compared to a single capillary and can be referred as to a massive improvement[S7].

It is also possible to use optics with specially designed capillary bundles having a small number of capillaries (optimally one) surrounded by solid fibers. Similar but "inverted" structures are used frequently in photonic crystal fibers. The use of "doped" solid rods (for suppression of a direct synchrotron beam) in polycapillary optics has been already demonstrated[S8]. The optimal diameter of such a bundle is slightly bigger than the focal spot dimension $\sim 2 \Delta x$. For an optics with a 1 mm diameter, consisting of 50% such bundles and of 50% normal capillary bundles, the number of "defects" will be $n_d \approx 3 \times 10^{-3}$ (now $n_d$ denotes the fraction of sparse capillaries) and $N_B \approx 1$ again giving a gain $G_{\text{SNR}} > 10$. Such a device will be especially suitable for multimodal absorption, phase-contrast and XRF imaging. Resolution could be improved by using capillaries with smaller channels. The drop in $\eta$ for smaller channels could be compensated by using optics with the same angular aperture but shorter input focal length and smaller geometrical aperture. SNR can be also increased by using optics with shorter focal length (it scales with $f^{-2}$) but working distances below 1 mm could be impractical.

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[S3] This terminology is adopted from x-ray and electron diffraction on imperfect crystals. For a very recent example of x-ray diffuse scattering see K. Ayyer et al. Nature 530, 202 (2016).
[S4] In $\mu$XRF such a gain is calculated relative to a small pinhole. However, in the context of this work the comparison with a single tapered capillary is more intuitive and reliable.