Charge and isospin symmetry breaking via external $\pi^0 - \eta$ meson mixing

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Abstract

A simple model of isospin and charge symmetry breaking due to external $\pi^0 - \eta$ meson mixing is presented. It is based on amplitudes extracted from the available experimental data for $pd \rightarrow ^3\text{He} \pi^0$, $pd \rightarrow ^3\text{He} \eta$ and $dd \rightarrow ^4\text{He} \eta$ reactions. Predictions of the strength of isospin symmetry breaking in $pd \rightarrow ^3\text{H} \pi^+ / ^3\text{He} \pi^0$ reactions are given. Within the same model, the cross section for the charge symmetry breaking $dd \rightarrow ^4\text{He} \pi^0$ reaction is calculated. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

Isospin symmetry and charge symmetry are not exact and in QCD they are broken due to the mass differences between up and down quarks and due to their electromagnetic interaction. It is believed that all isospin symmetry breaking (ISB) and charge symmetry breaking (CSB) effects that may be observed for hadronic systems have the same origin. They are, however, hidden in the secondary effects of the symmetries breaking which cause the difference between various hadron parameters as: hadron masses and coupling constants. Various processes have been studied searching for ISB and CSB effects (see Ref. [1] for a comprehensive review). However, there are usually significant problems with the interpretation of the observed deviations from the results predicted by symmetries. Only in some processes may direct symmetry breaking effects due to quark-mass differences be observed in hadronic systems. These studies are of special importance since they may allow a connection between the underlying QCD theory and hadronic systems in the nonperturbative regime. The observation of meson mixing offers the best possibility to study direct CSB effects. It occurs for mesons belonging to the same $SU(3)$ multiplets and is caused by the mass terms of the QCD hamiltonian. In the low mass hadron region,
the most important are $\pi^0 - \eta - \eta'$ and $\rho - \omega$ meson mixing. The observation of $\rho - \omega$ mixing is most straightforward since their masses are very close and, due to their large widths, these mesons overlap. Therefore their mixing may be observed directly in some reactions on the mass shell. Such studies were performed for the $e^+e^- \rightarrow \pi^+\pi^-$ reaction, showing interference behaviour due to $\rho - \omega$ mixing [2]. The $\pi^0 - \eta - \eta'$ mixing cannot be studied in such a direct way, due to large differences in meson masses. However evidence for their mixing was found in the decays of $\eta'$ [3], $\psi$ [4,5] and $\psi'$ [6,7].

Various theoretical estimations of the $\pi^0 - \eta - \eta'$ mixing strengths have been performed. Similar values of the $\pi^0 - \eta$ mixing angle equal to $\theta_m = 0.015$ rad were obtained from models based on QCD [8–11], while the $\pi^0 - \eta'$ mixing angle differs by a factor of two. The analysis of $\eta$ and $\eta'$ neutral decays leads to values for the mixing angle of 0.021 [12] or 0.015 [13]. More recent calculations based on chiral perturbation theory give $\theta_m = 0.015$ [14] and one using QCD sum rules yields $\theta_m = 0.014$ [15]. The lowest-order chiral perturbation theory [16] leads to a value of $\theta_m = 0.010$. This calculation does not include the $\pi^0 - \eta'$ mixing, so that the reported value may increase by about 30% [8]. The most recent calculation of the mixing angle is based upon an evaluation of quark loop diagrams with an up-down constituent quark mass difference of about 4 MeV [17]. Using this result a mixing angle $\theta_m = 0.014$ is obtained applying for the phenomenological $\eta - \eta'$ mixing angle a value of 0.733 rad in the strange–nonstrange basis [18].

The direct observation of $\pi^0 - \eta$ mixing may be achieved in some hadronic reactions. One of the most suitable is the charge-symmetry breaking $dd \rightarrow ^4\text{He} \pi^0$ reaction, since it is not influenced by electromagnetic effects. As was pointed out in Ref. [19], the meson mixing parameter may be extracted directly by comparing the cross section for this reaction with the $dd \rightarrow ^4\text{He} \eta$ reaction measured at the same beam energy. For a long time, various attempts to measure the $dd \rightarrow ^4\text{He} \pi^0$ reaction were undertaken leading only to upper limits for the cross section (see Ref. [20] and references therein). This reaction was observed in only one experiment at an incident deuteron energy of 1100 MeV [21] with a differential cross section of $d\sigma/\mbox{d }\varOmega = 0.97 \pm 0.20 \pm 0.15 \mbox{ pb/sr. However, this result was questioned even by some participants of the experiment (see quotation 6 in Ref. [22]). Also, a recent analysis [23] of the background reaction $dd \rightarrow ^4\text{He} \gamma \gamma$ shows that under the experimental conditions of Ref. [21], it may be confused with the CSB reaction.}

The effect of external $\pi^0 - \eta$ mixing may be also observed as an isospin symmetry breaking in the cross section ratio for $pd \rightarrow ^3\text{H} \pi^+ / ^3\text{He} \pi^0$ reactions. As was suggested in Ref. [22], this effect may be very pronounced for large relative pion-proton angles and at beam energies corresponding to the $pd \rightarrow ^3\text{He} \eta$ reaction threshold. Since the measurement of the ratio of cross sections may be performed with high accuracy, the observation of the $\pi^0 - \eta$ mixing in these reactions should be much easier.

A few measurements of this cross section ratio have been performed [24–27]. All measurements were done at single beam energies in the range from 450 MeV to 800 MeV. The ratio of cross sections was obtained for the same emission angles or at the same four momentum transfer. The reported values of the ratio are in the range of 2.17–2.36. Very recent measurements of $pd \rightarrow ^3\text{H} \pi^+ / ^3\text{He} \pi^0$ reactions were performed for a few beam...
energies around 300 MeV in a broad angular range [28]. Interpolation to the same four momentum transfer yields an average ratio very close to 2. Simple estimations [29] of various effects influencing the ratio leads to the conclusion, that the most important factor comes from the difference in the three nucleon wave functions, changing the predicted ratio to a value of about 2.15. This value coincides with experimental results within their errors. Isospin symmetry was investigated also in charge symmetric processes $nd \rightarrow ^3\text{He}\pi^-/{^3}\text{H}\pi^0$ at a few neutron energies in the range of 350–560 MeV [30] leading to a ratio of cross sections of 1.76. In this case the correction due to difference in the three nucleon wave functions tends to decrease the ratio. These measurements suggest an independence of the cross section ratio on the transferred momentum.

2. Phenomenological model

The present work was motivated by the ISB and CSB experiments planned at COSY Jülich [31]. The present simple model allows a prediction of the accuracy that experiments must reach in order to observe the symmetry breaking effects. Such a model was used in Ref. [19] in order to predict the cross section for the $dd \rightarrow ^4\text{He}\pi^0$ reaction. Later, it was extended taking into account the final state $^4\text{He}\eta$ interaction in the intermediate state [22,32] and attempting to explain the cross section for the $dd \rightarrow ^4\text{He}\pi^0$ reaction from Ref. [21].

The model is based on the assumption that in the first step of the reaction, the $\eta$ meson is produced, which turns into $\pi^0$ via meson mixing. The matrix element $\langle \eta|H|\pi^0 \rangle$ responsible for that transition may be related directly to the mixing angle

$$\theta_m = \frac{\langle \eta|H_{m,\text{em}}|\pi^0 \rangle}{m_{\pi^0}^2 - m_{\eta}^2},$$

where $m_{\pi^0}$ and $m_{\eta}$ are meson masses, and $H_{m,\text{em}}$ is the part of the hamiltonian containing the quark-mass terms and their electromagnetic interaction. For the reaction

$$dd \rightarrow ^4\text{He}\eta \rightarrow ^4\text{He}\pi^0$$

proceeding via intermediate state $^4\text{He}\tilde{\eta}$, the transition amplitude $T_{\eta^0}$ and the cross section $d\sigma/d\Omega_{\eta^0}$ may be expressed in terms of the transition amplitude $T_{\tilde{\eta}}$ and the cross section $d\sigma/d\Omega_\eta$ for the $dd \rightarrow ^4\text{He}\eta$ reaction and the $\pi^0-\eta$ mixing angle:

$$T_{\eta^0} = \theta_m \cdot T_{\tilde{\eta}},$$

$$\frac{d\sigma_{\eta^0}}{d\Omega} = \frac{p_{\eta^0}}{p_{\tilde{\eta}}} \cdot |T_{\eta^0}|^2 = \frac{p_{\eta^0}}{p_{\eta}} \cdot \theta_m^2 \cdot \frac{d\sigma_{\eta}}{d\Omega},$$

where the ratio of c.m. momenta is the phase-space factor and, for small mixing angles, the approximation $\tan \theta_m \approx \theta_m$ has been used. For the intermediate state the amplitude extracted from the measured cross section for the $dd \rightarrow ^4\text{He}\eta$ reaction may be used. For beam energies below the $\eta$-production threshold, this amplitude has to be extrapolated.

The effect of meson mixing should also influence the ratio of the cross sections for $pd \rightarrow ^3\text{H}\pi^+/{^3}\text{He}\pi^0$ reactions. While the mixing appears only in the $^3\text{He}\pi^0$ outgoing
channel, one may expect a deviation of this ratio from the value of 2 predicted by isospin symmetry. In order to find the ratio of these cross section, it is assumed that $\pi^0$ meson is produced via two intermediate states:

$$pd \rightarrow \begin{cases} \ ^3\text{He}\pi^0 \rightarrow ^3\text{He}\pi^0, \\ ^3\text{He}\eta \rightarrow ^3\text{He}\pi^0, \end{cases} \quad pd \rightarrow ^3\text{H}\pi^+, \quad$$

while the reaction $pd \rightarrow ^3\text{H}\pi^+$ proceeds without intermediate states. In a good approximation, the amplitudes for $^3\text{H}\pi^+$ and $^3\text{He}\pi^0$ are related as:

$$T_{\pi^+} = \sqrt{2} \cdot T_{\pi^0},$$

if one neglects the different electromagnetic effects for $^3\text{H}\pi^+$ and $^3\text{He}\pi^0$. The factor $\sqrt{2}$ comes from the isospin Clebsh–Gordon coefficients. The amplitude $T_{\pi^0}$ may be approximated by amplitude extracted from measured cross section for the $pd \rightarrow ^3\text{He}\pi^0$ reaction. The amplitude for $^3\text{He}\eta$ may be extracted from the measured cross section for the $pd \rightarrow ^3\text{He}\eta$ reaction and then extrapolated to the energies below $\eta$ threshold. Then the ratio of the cross sections for $^3\text{H}\pi^+$ and $^3\text{He}\pi^0$ is calculated as

$$R_{\pi^+} = \left( \frac{d\sigma_{\pi^+}/d\Omega}{d\sigma_{\pi^0}/d\Omega} \right) = \frac{p_{\pi^+}}{p_{\pi^0}} \frac{|T_{\pi^+}|^2}{|T_{\pi^0}|^2} = \frac{2}{p_{\pi^0} \left( |T_{\pi^0}|^2 + 2 \theta_{\pi^0} \right) \cos \phi},$$

where the amplitudes $T_{\pi^+}$ and $T_{\pi^0}$ are extracted from the corresponding reaction cross sections and $\phi$ is the relative phase of these amplitudes which cannot be obtained from measurements and will be a free parameter of the present model. The factor $p_{\pi^+}/p_{\pi^0}$ corrects for the different phase space caused by different pion masses.

The effects of ISB and CSB are here discussed at beam energies close to the $\eta$-production threshold. The square of the transition amplitude $|T_{\pi^0}|^2$ was fitted with a polynomial using the cross section for the $pd \rightarrow ^3\text{He}\pi^0$ reaction measured in the appropriate energy range at large and small $\theta_{\pi-\pi}$ relative angle [33–35]. The data with the corresponding fit are shown in Fig. 1.

In order to obtain the $\eta$ production amplitude, the experimental data for the $dd \rightarrow ^4\text{He}\eta$ reaction from Refs. [36,37] were used, while for the $pd \rightarrow ^3\text{He}\eta$ reaction, the data of Ref. [38] were taken. Two models for the calculations of the $\eta$ production amplitude were used. The first one uses the standard expression for the transition amplitude for the channel with strong final state interaction:

$$T_{\eta} = N \frac{a(\eta A)}{1 - ip_{\eta}a(\eta A)},$$

where $a(\eta A)$ stands for the scattering length for $\eta$ or $^3\text{He}$ or $^4\text{He}$ and $N$ is a normalization factor. The normalization factor was fitted in order to reproduce the available experimental data close to the reaction threshold. The results of this parametrization are shown in Fig. 2 as a dashed line together with the experimental data. The scattering lengths $a(\eta^3\text{He}) = (-3.8 + i 1.6)$ [38] and $a(\eta^4\text{He}) = (-2.2 + i 1.1)$ [37] were used. In order to extrapolate the amplitudes to the $\eta$ subthreshold region, a complex momentum $p_{\eta}$ was used, as proposed in Ref. [22].

Fig. 1. The square of the $pd \rightarrow ^3\text{He}~\pi^0$ amplitude for large ($\theta_{p-\pi} = 180^\circ$) and small ($\theta_{p-\pi} = 0^\circ$) proton-pion relative angles at beam momenta close to the $pd \rightarrow ^3\text{He}~\eta$ threshold. The solid lines represent the polynomial fit used for parametrization of the amplitudes. Data points are from Refs. [33–35].

Fig. 2. The square of the $pd \rightarrow ^3\text{He}~\eta$ and $dd \rightarrow ^4\text{He}~\eta$ amplitudes for beam energies close to threshold. The solid lines correspond to calculations using the $\eta$ amplitude from model of Ref. [39], while dashed lines are calculated according to Eq. 6 with $a(\eta ^3\text{He}) = (-3.8 + i 1.6)$ and $a(\eta ^4\text{He}) = (-2.2 + i 1.1)$. Data points for $^3\text{He}~\eta$ are from Ref. [38] and for $^4\text{He}~\eta$ from Refs. [36,37].

A more refined model for calculations of the $\eta$ production amplitude was proposed in Ref. [39]. It is based on multiple scattering theory and the $\eta$-nucleon $a(\eta N)$ scattering length is used as the input parameter. Here, all the parameters as calculated in Ref. [39] with $a(\eta N) = (0.281 + i 0.360)$ were used; only the normalization factor was fitted in order to describe all currently available data. The results of this model are also presented.
in Fig. 2 as a solid curve. The extrapolation to the $\eta$-subthreshold region was performed using a bound-state wavefunction for separable potentials with the Yamaguchi form factor [40]. This procedure is a natural extension of the method of Ref. [39], where the $\eta$-nucleus wavefunction was calculated with the same potential. The normalization of the bound-state wavefunction was obtained by requiring that amplitudes obtained above and below threshold give the same result at the threshold.

Both models for the $\eta$-production amplitude describe the available data satisfactorily, as is seen from Fig. 2. Some discrepancy exists for $^3$He $\eta$, and thus it will also appear in the amplitudes after extrapolation to the subthreshold region.

The first method applied for the calculations of the $\eta$ production amplitude neglects the off-shell variation of this amplitude. The approximate off-shell behaviour is included in the second model used for calculations of $T_\eta$. Therefore one may expect that the model of Ref. [39] delivers more appropriate values for this amplitude.

In the present model the mixing is dominated by real or nearly-real $\eta$ mesons produced close to the threshold in S-wave only; therefore the corresponding amplitude is isotropic. This leads to the isotropic cross section for $dd \rightarrow ^4$He $\pi^0$ reaction. This allows also to expect that deviations of the ratio $R$ from the value 2 should be larger at a proton–pion relative angle of $\theta_{\pi-\eta} = 180^\circ$, where the $|T_{\pi^0}|^2$ is small.

3. Results of the calculations

We now make use of the amplitudes fixed by experimental data to calculate the ratio of the cross sections $R$ for excess energies $\varepsilon$ ranging from $-10$ to $10$ MeV with respect to the $\eta$ threshold. In the calculations, a mixing angle $\theta_m = 0.015$ (as suggested by many model calculations) was used. The relative phase between $T_{\pi^0}$ and $T_\eta$ amplitudes was chosen to be $\phi = \pi$. For smaller values of the phase, the effect of the ISB will be smaller approaching zero at a value $\phi = \pi/2$. Then the ISB effect increases again with the phase decreasing to $\phi = 0$. The results of the calculations using the two approximations for the $T_\eta$ amplitude are shown in Fig. 3 for large and small proton–pion relative angles. Both methods lead to very similar predictions for $\varepsilon \geq 0$, while some discrepancy is observed in the subthreshold region. It is seen that the largest ISB effect appears at the $\eta$-threshold, where the ratio is $R = 2.4$ for $\theta_{\pi-\eta} = 180^\circ$ and $R = 2.03$ for $\theta_{\pi-\eta} = 0^\circ$. Therefore, only a measurement with a large relative proton–pion angle has a chance to observe an ISB effect. It should be pointed out that the differences in the $^3$H and $^3$He wave functions may also change the ratio $R$; however in the very small energy region of present interest, this change will be almost energy independent. For the beam energy varying by about 40 MeV the momentum transfer changes by about 10 MeV/c only. In the impulse approximation this corresponds to very small changes in the three body wave function. It may be expected that at high energy the multistep processes would dominate over impulse approximation diagrams. Their presence will smear out the dependence of the ratio on the details of the three body wave function. Therefore, the crucial point for discovering an ISB effect is the measurement of beam energy dependence of the ratio $R$. In this way, the interference
Fig. 3. The calculated ratio of the cross sections for $pd \rightarrow ^3\text{H}\pi^+/^3\text{He}\pi^0$ reactions as a function of energy excess $\varepsilon$ with respect to the $\eta$ threshold in the $pd \rightarrow ^3\text{He}\eta$ reaction. The solid curves correspond to calculations using an $\eta$ amplitude extracted within the model of Ref. [39] while dashed curves are calculated according to Eq. 6 with $a(\eta^3\text{He}) = (-3.8 + i1.6)$. The mixing angle $\theta_m = 0.015$ was assumed. The upper part is for large ($\theta_{p-\pi} = 180^\circ$) and the lower part is for small ($\theta_{p-\pi} = 0^\circ$) proton–pion relative angle.

behaviour of the ratio as predicted by the present model should be easily distinguished from electromagnetic effects. The discussed method of ISB measurement will be almost immune on the details of the reaction mechanism and the precise theoretical interpretation.

We now make use of the $T_\eta$ amplitude fixed by the experimental $dd \rightarrow ^4\text{He}\eta$ cross section with a mixing angle $\theta_m = 0.015$ to calculate the cross section for the $dd \rightarrow ^4\text{He}\pi^0$ reaction. The results are presented in Fig. 4. Both models for the $\eta$ amplitude give almost the same predictions for the $^4\text{He}\pi^0$ cross section as a function of excess energy $\varepsilon$ in the range from $-10$ to $10$ MeV in respect to the $\eta$ threshold for $dd \rightarrow ^4\text{He}\eta$ reaction. Even the questionable experimental value at subthreshold energy $\varepsilon = -8.76$ MeV is reproduced.

This is compatible with results of Refs. [22,32]. However, no discrepancy is observed even applying completely different models for the $\eta$-transition amplitude. The cross section for $^4\text{He}\pi^0$ is predicted to be largest at the $\eta$ threshold and reaches a value of $4.4$ pb/sr there.

Finally, the dependence of the ratio $R$ and of the cross section for the $dd \rightarrow ^4\text{He}\pi^0$ reaction close to the $\eta$ threshold on the mixing angle was calculated using $\eta$ amplitudes derived within the model of Ref. [39]. The results of the model predictions are presented in Fig. 5. It is seen that ISB may be observed in ratio $R$ measurements even for a mixing angle $\theta_m = 0.006$, if the experimental accuracy is about 2%. In experiments reaching a sensitivity of $0.1$ pb/sr, even for a mixing angle $\theta_m = 0.004$ the CSB signal should be visible in the $dd \rightarrow ^4\text{He}\pi^0$ reaction. In order to reach such sensitivity, the background from the $dd \rightarrow ^4\text{He}\gamma\gamma$ reaction must be well under control. This may be achieved using a tensor-polarized deuteron beam and the fact that the $^4\text{He}\pi^0$ exit channel has maximum analysing power $t_{20}$ at an angle of $0^\circ$, although this does not hold for the background
Fig. 4. The calculated cross section for the $dd \rightarrow ^4\text{He}\pi^0$ reaction as a function of the excess-energy $\varepsilon$ for the $dd \rightarrow ^4\text{He}\eta$ reaction. The solid curve corresponds to the calculations using an $\eta$ amplitude from the model of Ref. [39] while the dashed curve was calculated according to Eq. (6) with $a(\eta^4\text{He}) = (-2.2 + i1.1)$. A mixing angle of $\theta_m = 0.015$ was applied. The data point is from Ref. [21].

Fig. 5. Upper part: the calculated ratio of cross sections for $pd \rightarrow ^3\text{H}\pi^+ / ^3\text{He}\pi^0$ reactions at large ($\theta_{p-p} = 180^\circ$) proton–pion relative angle as a function of the mixing angle $\theta_m$ for a beam energy corresponding to the $pd \rightarrow ^3\text{He}\eta$ reaction threshold. Lower part: the predicted cross section for the $dd \rightarrow ^4\text{He}\pi^0$ reaction as a function of the mixing angle $\theta_m$ for a beam energy corresponding to the $dd \rightarrow ^4\text{He}\eta$ reaction threshold. Solid curves correspond to the calculations using $\eta$ amplitudes derived within the model of Ref. [39].

A similar method of background subtraction was applied in the measurements of the $dd \rightarrow ^4\text{He}\eta$ reaction at threshold [37].

4. Summary

To summarize, we have used a simple model based on experimental transition amplitudes to calculate the effects of ISB and CSB that may be observed in $pd \rightarrow ^3\text{H}\pi^+ / ^3\text{He}\pi^0$ and $dd \rightarrow ^4\text{He}\pi^0$ reactions. The predicted deviations from the isospin and charge symmetry are independent of the method used to calculate the behaviour of the $\eta$ amplitude, unless the corresponding data for $\eta$ production close to threshold are properly
reproduced. In case of ISB investigated in $pd \rightarrow ^3H \pi^+ / ^3He \pi^0$ reactions, only the lower limit for the mixing angle may be obtained. This is due to the unknown phase of the two interfering processes. The mixing angle, however, may be obtained directly from the comparison of $dd \rightarrow ^4He \pi^0 / ^4He \eta$ reactions measured at the same energy. The predicted magnitude of ISB and CSB is reachable by the experiments planned at COSY Jülich [31].

References