Detailed comparison of the $pp \rightarrow \pi^+ pn$ and $pp \rightarrow \pi^+ d$
reactions at 951 MeV

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Abstract

The positively charged pions produced in proton–proton collisions at a beam momentum of 1640 MeV/c were measured in the forward direction with a high resolution magnetic spectrograph. The missing mass distribution shows the bound state (deuteron) clearly separated from the $pn$ continuum. Despite the very good resolution, there is no evidence for any significant
production of the \( pn \) system in the spin-singlet state. However, the \( \sigma(pp \rightarrow \pi^+pn) / \sigma(pp \rightarrow \pi^+d) \) cross section ratio is about twice as large as that predicted from \( S \)-wave final-state-interaction theory and it is suggested that this is due to \( D \)-state effects in the \( pn \) system.

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There is a very extensive literature on the \( pp \rightarrow \pi^+d \) reaction and many detailed analyses have been made [1], but much less is known about the production of the continuum in the \( pp \rightarrow \pi^+pn \) case. Data covering low excitation energies generally show the strong \( S \)-wave final-state-interaction (fsi) peak corresponding to the \( pn \) spin-triplet which has, as a characteristic energy scale, the binding energy of the deuteron \( (B_t = 2.22 \text{ MeV}) \). However, the energy resolution is generally insufficient to identify the analogous spin-singlet fsi peak, for which the corresponding energy scale is only \( B_s = 0.07 \text{ MeV} \) [2]. Indirect evidence suggests that spin-singlet production is much weaker than that of spin-triplet for medium energy proton beams [3], and this is confirmed by data from the isospin-related \( pp \rightarrow \pi^0 pp \) reaction, though these are limited in incident momentum or energy resolution [4]. Such weak spin-singlet production accords well with theory, because the influence of the \( \Delta \)-isobar is minimal there.

A useful way of trying to extract the spin-singlet contribution is through the comparison of the overall strengths of the cross sections for \( pn \) and deuteron final states. Using final-state-interaction theory, Fäldt and Wilkin derived the extrapolation theorem which relates the normalisations of the wave functions for \( S \)-wave bound and scattering states [5]. This has been exploited to predict the double-differential centre-of-mass (cm) cross section for the \( S \)-wave spin-triplet component in \( pp \rightarrow \pi^+pn \) in terms of the cross section for \( pp \rightarrow \pi^+d \) [6]:

\[
\frac{d^2\sigma}{d\Omega dx}(pp \rightarrow \pi^+|pn\rangle_s) = \frac{p(x)}{p(-1)} \frac{\sqrt{x}}{2\pi(x+1)} \frac{d\sigma}{d\Omega}(pp \rightarrow \pi^+d). \tag{1}
\]

Here \( x \) denotes the excitation energy \( \varepsilon \) in the \( np \) system in units of \( B_t \), \( x = \varepsilon/B_t \), and \( p(x) \) and \( p(-1) \) are the pion cm momenta for the \( pn \) continuum or deuteron, respectively.

In the derivation of Eq. (1) it is assumed [6] that the pion production operator is of short range and that \( x \) is not too large, so that the \( pn \) \( P \)-waves contribute little. Most critical though is the neglect of channel coupling through the \( pn \) tensor force, so that the equation could only be valid provided that the \( D \)-state effects are small in the production of both the bound state and continuum.

The fsi peak arises from the \( \sqrt{x}/(x+1) \) factor in Eq. (1) and there should be an analogous spin-singlet enhancement, where the deuteron binding energy \( B_t \) is replaced by the energy \( B_s \) of the virtual state in the \( S = 0, T = 1 \) system. At low excitation energies one therefore expects that

\[
\frac{d^2\sigma}{d\Omega dx}(pp \rightarrow \pi^+|pn\rangle_s) = \xi \left( \frac{\varepsilon + B_t}{\varepsilon + B_s} \right) \frac{d^2\sigma}{d\Omega dx}(pp \rightarrow \pi^+|pn\rangle_t), \tag{2}
\]

where we use the factor \( \xi \) to quantify the ratio of spin-singlet to spin-triplet production.

Since the best resolution in excitation energy so far achieved was typically \( \sigma = 350 \text{ keV} \) [7], any singlet peak would have been smeared significantly in all published data. However, by estimating the \( S \)-wave triplet contribution to the \( pp \rightarrow \pi^+pn \) cross section from Eq. (1) and subtracting it from the observed data, some measure for the singlet production could be obtained. In most experiments where only the \( \pi^+ \) was detected, the limited resolution did not guard against some leakage of the deuteron peak into the continuum region [8–10]. On the other hand, detecting the \( \pi^+ \) and proton in coincidence [11], while identifying well the continuum channel, loses the relative normalisation with the \( \pi^+d \) final state, which is so important in the implementation of Eq. (1). Therefore, in addi-
tion to the pion spectrum, Betsch et al. [12] measured coincidences between pion and proton, but then had to rely on Monte Carlo simulations. For 600 MeV and below, the data seemed to confirm that the singlet contributed at most 10% of the cross section, though at 1 GeV a higher figure was likely [9].

Most of the uncertainties mentioned above could be minimised by measuring simultaneously the whole pion spectrum, corresponding to both the $d$ and $pn$ final states, with a high resolution. One could then identify clearly any singlet peak and also separate unambiguously the $pp \rightarrow \pi^+ d$ from the $pp \rightarrow \pi^+ pn$ reaction. This was our primary goal when planning a new experiment. Pions were observed near zero degrees with the 3Q2D spectrograph Big Karl [13] at the COSY accelerator in Jülich. The setting of the magnetic field was such that the pions from the $pp \rightarrow \pi^+ d$ reaction were well within the acceptance of the spectrograph, thus avoiding the creation of background from the side yoke. Position and track direction of the pions in the focal plane were measured with two sets of multiwire drift chambers, each having six layers. The chambers were followed by scintillator hodoscopes that determined the time of flight over a distance of 3.5 m. In order to optimise the momentum resolution, a liquid hydrogen target of only 2 mm thickness was used with windows made of 1 μm Mylar [14]. The beam was electron cooled at injection energy, and, after acceleration, stochastically extracted. Electron cooling usually yields a lower beam intensity than for an uncooled beam. Both, electron beam cooling and the thin target, resulted in a small luminosity, thus making dead time corrections negligible. This gave an energy resolution of $\sigma = 97$ keV for the deuteron peak. This was much better than that found in a test run without beam cooling and, in particular, the background was considerably reduced.

The results of our experiment are shown in Fig. 1 as function of the excitation energy in the $pn$ system. Though corrections for acceptance, etc., have been included, yielding the same efficiency for both reactions; these, in fact, vary slowly with $\epsilon$ for energies below 20 MeV. Noting the logarithmic scale in the figure, it is clear that there is an excellent distinction between the $pp \rightarrow \pi^+ pn$ from the $pp \rightarrow \pi^+ d$ reactions. Since the luminosity and detection efficiencies largely cancel out between them, this means that we have a very good determination of the relative cross sections for $\pi^+ d$ and $\pi^+ pn$ final states.

Also shown in Fig. 1 is the prediction of the continuum production from the $S$-wave $f_{si}$ theory of Eq. (1) [6].
Fig. 2. Comparison of the measured $pn$ excitation energy spectrum on a linear scale with the prediction of Eqs. (1), (2) for the shape of the singlet cross section. The error bars contain a tiny contribution from the uncertainty in the acceptance correction.

the whole of the spectrum. This is in contrast to the TRIUMF data, taken a bit below the $\Delta$ resonance, for which the formula predicts reasonably the normalisation and shape of the spectra for $\epsilon < 15$–20 MeV [10]. On the other hand, it should be noted that, if our data are artificially degraded such that the resolution is the same as that achieved in the Leningrad experiment at the neighbouring energy of 1 GeV ($\sigma \approx 3$ MeV) [9], the two sets of results overlap very well. Nevertheless, the poor resolution allowed the authors of Ref. [6] to ascribe the factor-of-two discrepancy to the production of spin-singlet final states. We can, however, check this hypothesis independently by studying the shape of the missing-mass spectrum.

As is evident from Eq. (2), the cross section for producing a $pn$ singlet state must show a sharp spike just above threshold and, due to our good resolution, this prominent feature should remain even after convolution with this resolution. In Fig. 2 are shown the predictions of Eqs. (1), (2) with $\xi = 1$, modified by the inclusion by an extra factor of $(1 + \epsilon/E_s)$ to try to take into account deviations from the extrapolation theorem [15]. The value of $E_s = 24$ MeV is derived from the scattering length and effective range [16] though, by the point that this becomes significant, the $S$-wave ansatz is dubious. This is of little importance, there is no hint of any sharp needle in the data of Fig. 2 at low $\epsilon$ and, in fact, the shape of the cross section is completely compatible with pure spin-triplet production. Fits of Eq. (2) in the small $\epsilon$ region with free amounts of singlet and triplet show that $\xi < 10^{-4}$ at the one standard deviation level, and this corresponds to a practically vanishing fraction of the singlet part. As a consequence, we must seek elsewhere for the factor-of-two discrepancy between our data and the results of Eq. (1).

The deviation is unlikely to be due to the $pn$ system being at too high an excitation energy because there are problems already at $\epsilon = 3$ MeV. However, as has been stressed previously, the extrapolation theorem linking the bound and scattering wave functions is only valid if one can neglect completely $D$-state effects [5]. Though the $D$-state wave functions are suppressed at short distances by the centrifugal barrier, the $S$-wave is also reduced in this region by the repulsive core. Thus the $D$-state might be significant for pion production despite the relatively small probability in the deuteron, especially if $S$–$D$ interference terms are important.

We consider a microscopic calculation of the actual three-body $\pi^+pn$ final state reaction to be beyond the scope of the present work. Nevertheless, to investigate
the effects of the $D$-wave, at least semi-quantitatively, we have made estimations of the $pp \rightarrow \pi^+ d$ differential cross section following the formalism described in Ref. [17]. Using a standard deuteron wave function [18] with a normal $D$-state, this reproduces well the experimental data [1]. The calculations have, however, been repeated with a reversed sign for the $D$-state amplitude and also with no $D$-state at all. Now for kinematic reasons the $pn$ $D$-state scattering wave function must vanish like $\varepsilon^4$ as $\varepsilon \to 0$ so that its sign should change when going from the bound state (deuteron) to the continuum $pn$ pair [5]. One can therefore get an idea of the effect of the $D$-state in the continuum by using a deuteron wave function with the opposite sign for the $D$-wave.

The predictions for the forward cross section are shown in Fig. 3 as a function of the dimensionless pion cm momentum $\eta = p/m_{\pi^+}$, the present experiment corresponding to $\eta = 2.6$. The zero $D$-state calculation is approximately the average of the other two, showing that the effects are mainly due to $S-D$ interference. At high energies the inclusion of the $D$-state decreases the $pp \rightarrow \pi^+ d$ cross section and so we would expect it to increase the $pp \rightarrow \pi^+ pn$ continuum production rate. The converse is true at low energies, though the exact position of the cross-over point, here predicted to be at $\eta \approx 1.6 (T_p \approx 600$ MeV), could be model dependent. Nevertheless, we would certainly expect there to be a different influence of the $D$-state on either side of the $\Delta$ peak. Given the uncertainties in the estimation of the $pp \rightarrow \pi^+ d$ cross section and the simplistic way that we have used this to speculate on the influence of the $D$-wave on continuum production, the fact that the factor of 2.2 difference between the calculations with the changed sign of the $D$-state at $\eta = 2.6$ coincides exactly with the discrepancy between the data and the $S$-wave theory shown in Fig. 1 may be fortuitous. Close to or just below the resonance one would expect smaller deviations from the extrapolation theorem associated with the $D$-state, and this certainly seems to be the case experimentally [6–8,10]. To quantify the deviations would require further high resolution runs which could identify clearly the singlet production from the shape of the spectrum.

In summary, we have measured the missing mass spectrum from the $pp \rightarrow \pi^+ X$ reaction in the forward direction. Despite the rather high beam momentum of 1640 MeV/$c$, the excellent resolution allowed
the complete separation of the deuteron from \( \text{pn} \) continuum and also showed that the production of spin-singlet states was negligible at this momentum. Deviations from the results of \( S \)-wave \( fsi \) theory could be ascribed semi-quantitatively to the effects of the tensor force in the \( \text{pn} \) system and an extension of this to encompass the coupled \( S-D \) system would be of great help. It is also to be hoped that a full microscopic calculation of the three-body \( \pi^+ \text{pn} \) final state production will be undertaken to complement the two-body results quoted here [17]. This might then confirm our hypothesis of the great influence of the deuteron \( D \)-state in pion production above the \( \Delta \) resonance.

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