

**π⁰ - η meson mixing in pd→³Heπ⁺/³Heπ⁰ reactions**

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Beam momentum dependence of the experimental cross sections for pd→³Heπ⁺/³Heπ⁰ reactions have been studied at large proton-pion relative angle. The measurements were performed for five beam momenta in the region of the η meson production threshold. Observed behavior of the measured cross sections ratio reveals manifestations of the isospin symmetry breaking effects. A simple model allows one to interpret the isospin symmetry breaking in terms of π⁰-η meson mixing. Based on that model analysis of the experimental results leads to the mixing angle value of 0.006±0.005 rad.

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The isospin symmetry is an approximate symmetry observed in hadron properties and interactions. Regardless of its exactness the use of the isospin symmetry is fruitful in various nuclear and particle physics applications. On the hadronic level the isospin symmetry breaking could be attributed to the electroweak interaction, differences in the hadron masses and to some extent to the strong interaction. All these isospin symmetry breaking sources should find their explanation in QCD. On the quark level the isospin symmetry is broken due to the $u$ and $d$ quarks current mass difference [1] and their electroweak interaction. The electroweak interaction may be to some extent treated precisely for hadrons. It is, however, not easy to relate the isospin symmetry breaking on hadronic level to the quark mass difference. Since the $u$ and $d$ quark mass difference is very small [2] the isospin symmetry breaking is also small for hadronic systems, where the current masses are replaced by large constituent masses. In some part this quark mass difference is responsible for the hadron mass difference, which obviously leads to the isospin symmetry breaking on hadronic level. This effect can be partially described in the chiral perturbation theory or in the lattice calculations which lead to an estimation of the $u$ and $d$ quark mass ratio or their average mass (see Ref. [2] for review). Beside hadron mass differences, dynamical effects in the strong hadron interaction are also induced by the quark mass difference. One of the most important dynamical effects is the light meson mixing induced by the $u$ and $d$ quark mass difference which leads to isospin breaking hadronic strong forces.

A large interest in investigations of meson mixing is due to the fact that such dynamics can be calculated perturbatively and may be directly related to the quark mass difference. On the other hand this mixing may be easily studied experimentally via the isospin or charge symmetry breaking processes. The most important cases comprise π⁰-η⁻-η⁺ and ρ-ω meson mixing. The ρ-ω meson mixing can be observed directly in some reactions on the mass shell since the masses and widths of these mesons cause their strong overlapping. Indeed, a strong interference due to ρ-ω mixing was observed in the $e^+ e^- → π^+ π^- ρ$ reaction [3]. The observation of π⁰⁻-η⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻˓
The magnitude of isospin or charge symmetry breaking is described by the meson mixing angle $\theta_m$. Using the orthogonality condition for real mesons it is possible to relate the mixing angle to the matrix element $\langle \pi^0 | H | \eta \rangle$ that can be calculated on the quark level, obtaining

$$\theta_m = -\frac{\langle \pi^0 | H | \eta \rangle}{m_{\pi^0} - m_\eta^2}.$$  

(2)

A measurement of the $\pi^0$-$\eta$ mixing angle may deliver additional limits on the $u$ and $d$ quark masses since it is sensitive to their mass difference. Magnitude of this angle may be important in the analysis of CP violation sources leading to the enhancement of $e^+e^-$ [4,5]. The isospin symmetry breaking due to $\pi^0$-$\eta$ mixing has strong implications on the analysis of $B \to \pi \pi$ decays [6] and impacts strongly the extracted value of $\sin 2\alpha$ which is related to certain CKM matrix elements.

Various theoretical attempts to calculate the $\pi^0$-$\eta$-$\eta'$ mixing with QCD based models agree in results for the $\eta'$-$\eta'$ mixing angle. Also predictions for the $\pi^0$-$\eta$ mixing angle are similar and deliver values of 0.014–0.015 rad (see Ref. [7] for references). There are, however, some calculations delivering values out of this range, e.g., 0.034 rad [8] or 0.010 rad [9]. Experimental evidence comes from isospin or charge symmetry forbidden meson decays which may be interpreted in the meson mixing formalism (see Ref. [10] for references). Charge symmetry breaking was observed also in hadronic reactions $\pi^+d \to pp\eta$ and $\pi^-d \to nn\eta$, which, interpreted within a model for $\eta$ production, delivered the $\pi^0$-$\eta$ mixing angle of 0.026$\pm$0.007 rad [11].

The $\pi^0$-$\eta$ meson mixing may play an important role also in the $pd \to ^3H\pi^+/^1H^0\eta^0$ reactions in a specific beam momentum region [7]. Experimentally the isospin symmetry breaking may be inspected by observing the ratio $R$ of the differential cross sections for these reactions:

$$R = \frac{\frac{d\sigma}{d\Omega}(pd \to ^3H\pi^+)}{\frac{d\sigma}{d\Omega}(pd \to ^3He\pi^0)}.$$  

(3)

The exact isospin symmetry predicts this ratio to be equal to 2. This result is obtained for the $\pi^0$ meson treated as the pure isospin 1 state. Due to meson mixing the real $\pi^0$ mesons have a small admixture of isospin 0, which leads to isospin symmetry breaking. Since this admixture is very small, the isospin symmetry breaking effects are also very small. However, under special conditions they may be enhanced, allowing a direct investigation of the $\pi^0$-$\eta$ meson mixing. In the $pd \to ^3H\pi^+/^1H^0\eta^0$ reactions a large effect of isospin symmetry breaking due to meson mixing should appear for beam momenta close to the $\eta$ meson production threshold and for outgoing products' angles corresponding to large relative proton-pion angles. The effect can be observed in the ratio of the cross sections for the $^3$H and $^3$He production. The predicted deviations of this ratio from 2 are about 20% [7].

In order to extract the mixing angle, the beam momentum dependence of the ratio $R$ has to be measured over a narrow momentum region close to the $\eta$ production threshold. Changes of the ratio will indicate isospin symmetry breaking due to the $\pi^0$-$\eta$ meson mixing. Besides the meson mixing effect the isospin symmetry might be broken due to the electromagnetic interaction and to the mass difference of the outgoing particles. However, the electromagnetic interaction cannot produce strong variation of the ratio over the small beam momentum range, its influence would eventually result in only a constant shift of the ratio $R$. Since the investigations are performed for beam momenta far above the pion production threshold, the mass differences of outgoing particles introduce negligible effect on the isospin symmetry breaking.

Few measurements of the differential cross sections of the $pd \to ^3H\pi^+/^1H^0\eta^0$ reactions for large relative proton-pion angles have been performed up to now, at several beam momenta [12–14]. These measurements addressed separately $^3$H$\pi^+$ and $^3$He$\eta^0$ outgoing channels. Some of them were performed at the same incident momentum, therefore allowing one to calculate the ratio $R$. The resulting values vary in the range of 1.3–2.4. These values substantially differ from the expected value of 2, suggesting that the quoted systematic errors of a few percent are strongly underestimated. In order to reduce the systematic uncertainty, a simultaneous measurement is necessary. Then the ratio of the differential cross sections may be obtained directly from the measured numbers of counts $N_{^3H}$ and $N_{^3He}$ for the detected $^3$H and $^3$He, respectively:

$$R = A \frac{N_{^3H}}{N_{^3He}},$$  

(4)

where $A$ is the relative acceptance of the detection systems where $^3$H and $^3$He are registered. In such a case the main and virtually the only relevant systematic uncertainty comes from the relative acceptance measurement.

We have designed and performed the first measurement of the ratio $R$, based on the simultaneous detection of both reaction products. In this way large uncertainties connected with absolute normalization of two cross section values are avoided if only the ratio is concerned. The experiment was performed at the COSY accelerator [15] in Jülich using an extracted beam with an intensity of $5 \times 10^8$ protons/s. The ratio of differential cross sections was measured for five beam momenta: 1.56, 1.57, 1.571, 1.59, and 1.70 GeV/$c$ with the beam momentum determination accuracy better than 0.1%. A liquid deuterium target [16] with thickness of 1 cm was used, yielding a luminosity of about $3 \times 10^{31}$ s$^{-1}$ cm$^{-2}$. The reaction products were detected using the upgraded Big Karl magnetic spectrometer. In the measurements the $^3$H and $^3$He ejectiles with maximal momenta were registered, which corresponded to a large proton-pion relative angle. The individual cross sections for the $pd \to ^3H\pi^+$ and $pd \to ^3He\pi^0$ reactions were also obtained,
however, with larger systematical uncertainties. More details on the experimental procedure and on data evaluation can be found in Refs. [17,18]; below, only the most important features will be presented.

The outgoing $^3$H and $^3$He particles have almost the same momenta, but due to different charges their magnetic rigidities differ by a factor of 2. The upgraded Big Karl spectrometer is capable to detect simultaneously particles with such different magnetic rigidities. The $^3$He ejectiles are bent to the standard focal plane while $^3$H particles reach the detection system located at the first dipole outer yoke. Both detection systems contain drift chambers used for particle tracking and two planes of the scintillating hodoscopes delivering information on the energy losses and on the time of flight. All these data are sufficient to unambiguously identify the investigated reactions. At the focal plane the specific energy losses and the time of flight information are used for particle identification. The data on the particle track are used to determine the particle momentum, enabling one to calculate the missing mass. Finally, the missing mass spectrum is obtained from the reconstructed tracks was used first. The events associated with the particles of well defined momentum are correlated when regarding their emission angles and detection positions. Therefore, projecting the events on the angle vs position spectra for horizontal and vertical directions results in formation of narrow bands for ejectiles with specific momenta. Additionally it was requested that the energy loss exceeds some threshold value appropriate for the $^3$H ejectiles. With all these cuts the time of flight spectrum was constructed.

The background in the $^3$H spectrum is substantial when compared to that observed for the $^3$He ejectiles. The main source of this background comes from the secondary particles produced by the beam hitting the first dipole yoke. This beam dump is located close to the detection system at the first dipole exit since the $^3$H and beam momenta are very similar. In order to reduce the background to an acceptable level a nonzero beam incidence angle on the target has been introduced. It was achieved by a common action of the steerer magnets in the beam line and a special dipole magnet, located close to the target. This magnet arrangement bent the beam in front of the target tilting it horizontally with respect to the spectrometer optical axis. The beam incidence angle on the target was precisely calibrated using the $pp \rightarrow d \pi^+$ reaction registered with the standard focal plane detection system. Since the missing mass is sensitive to the beam angle at the target, it was used to calibrate the beam bending dipole current. Later on, in all measurements the beam incidence angle on the target was kept constant at the value of 15.3 ± 0.5 mrad.

The relative acceptance measurement was also performed with the use of the $pp \rightarrow d \pi^+$ reaction. At the beam momentum of 1.206 GeV/c the ratio of deuteron and pion momenta is equal to 2. Therefore, their simultaneous detection simulates exactly the conditions for $^3$H and $^3$He registration. The kinematical coincidences of outgoing deuterons and pions were used for the relative acceptance A determination, leading to the value $A = 18.8 \pm 2.0$. Due to certain experimental limitations [17], the value of $A$ obtained in this method might be considered as an upper limit for the relative acceptance. To check the result, the relative acceptance measurements based on the equality of the center of mass cross sections for deuterons emitted at 0° and 180° were performed for each beam momentum. It has been found that this procedure leads to similar results for the relative acceptance. However, since the ratio of the deuteron momenta is not exactly equal to 2, it is necessary to introduce some corrections, leading to large systematical uncertainties. Therefore finally the $A$ value obtained in the coincidence measurement was used to determine the cross sections ratio and individual values of the $pd \rightarrow ^3$H$\pi^+$ reaction cross section.

The number of events corresponding to the detected $^3$He from the $pd \rightarrow ^3$He$\pi^0$ reaction was extracted from the missing mass spectra by fitting the Gaussian distribution together with a linear background. Then the events were integrated within the ±3σ region and the background function obtained from the fit was integrated in the same range. The final number of events for $^3$He, $N_{^3He}$, was obtained as the difference of these two integration results, with the statistical error reflecting the statistics of both, the peak and the background.

The number of events corresponding to the detected $^3$H from the $pd \rightarrow ^3$H$\pi^+$ reaction was extracted from the time of flight spectra. The particles of a given momentum not originating from the target are lying outside the correlation band in the position-angle spectrum for vertical direction. Using gates positioned in the regions above and below this correlation band, the background distribution can be obtained. It has been found that the background is almost identical for events lying above and below the correlation band; it was, however, approximated by the average of these two. Such a background was subtracted from the time of flight spectra and the final number of $^3$H events, $N_{^3H}$, was obtained by integrating the fitted to the triton peak Gaussian distribution within the ±3σ region. Separately, the background distribution was integrated within the same region in order to evaluate the total statistical uncertainty of the final number of $^3$H events.

The resulting numbers of events for $^3$H and $^3$He were corrected for the corresponding detection efficiencies of the two detection systems. The total efficiency is given by the drift chambers’ efficiency and the events rejection probability due to two hits in a single scintillator paddle. The drift chambers’ efficiencies were varying from run to run due to different high voltages and slightly different gas mixtures. Their values have been determined with a good accuracy for every beam momentum, giving a correction factor of 85%–98%. The double hit correction was important only for the triton detection system, where its value was found to be 91%–97%. It should be noticed that the total efficiencies are known with the relative precision of below 0.5%.

Using the obtained final numbers of events for $^3$H and $^3$He and the relative acceptance of the two detection systems for these two ejectiles, the ratio of the cross sections was calculated for every beam momentum. The results, together with their pure statistical errors, are presented in Table I and are shown in Fig. 1(c). The ratios of the cross sections cal-
TABLE I. Experimental center of mass cross section values (in nb/sr) for the $pd\rightarrow^3\text{He}\pi^+/^3\text{H}\pi^0$ reactions for the relative proton-pion angle of 180° and their ratios for five beam momenta. Only statistical errors are listed.

<table>
<thead>
<tr>
<th>$p_{lab}$ [GeV/c]</th>
<th>$d\sigma/d\Omega$ ($^3\text{He}$)</th>
<th>$d\sigma/d\Omega$ ($^3\text{H}$)</th>
<th>Ratio R</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.560</td>
<td>47.4±3.9</td>
<td>23.1±0.6</td>
<td>2.05±0.17</td>
</tr>
<tr>
<td>1.570</td>
<td>45.8±6.6</td>
<td>24.9±0.7</td>
<td>1.84±0.27</td>
</tr>
<tr>
<td>1.571</td>
<td>47.5±2.2</td>
<td>21.1±0.2</td>
<td>2.24±0.11</td>
</tr>
<tr>
<td>1.590</td>
<td>62.6±8.6</td>
<td>24.3±1.0</td>
<td>2.57±0.37</td>
</tr>
<tr>
<td>1.700</td>
<td>44.0±4.7</td>
<td>17.8±0.4</td>
<td>2.47±0.27</td>
</tr>
</tbody>
</table>

We have calculated also the absolute values of the cross sections for the two investigated reactions, using the normalization to the target thickness and beam intensity, the appropriate solid angles, and applying proper dead time corrections. Target thickness is known with an accuracy of 3% from the dimensions of the cylinder containing the liquid deuterium. The beam intensity was measured with calibrated luminosity monitors mounted close to the target. Their calibration was performed with several precisely known beam intensities reduced so that the direct beam could have been detected at the first dipole exit detection system. The linear dependence of the counting rate in the luminosity monitors as the function of the full beam intensity was extrapolated to the high beam intensities used in the regular measurement. This procedure leads to the systematical uncertainty in the cross sections determination of 5%. The known optical transformation parameters for the tracks detected at the focal plane were used to determine the angles of the ejectiles emitted from the target. This allowed us to calculate the solid angle for the detection system at the focal plane with the precision of 7%. As already mentioned, the accuracy of the acceptance of the first dipole exit detection system was measured relative to the focal plane acceptance with an accuracy of 10%. The dead time corrections were determined with precision better than 1%. All systematical uncertainties yield the total systematical errors of 9% and 13.5% for the absolute values of the cross sections for $^3\text{He}$ and $^3\text{H}$, respectively. The results of the center of mass cross sections for the two investigated reactions are given in Table I and are shown in Figs. 1(a) and 1(b), together with the existing data [12–14].

The absolute values of the cross sections measured for the $pd\rightarrow^3\text{He}\pi^+/^3\text{H}\pi^0$ reactions agree very well with the existing data. Also the ratios of the cross sections are in good agreement with those calculated from the available data. The measured values of the cross section ratio indicate the existence of isospin symmetry breaking effects in the region of the $\eta$ meson production threshold. It should be pointed out, however, that already the ratio of the cross sections calculated from the data of Refs. [13,14] shows large deviations from the expected value of 2 even in the beam momentum region far below the $\eta$ meson production threshold. The present data lead to the ratio values larger than 2 at beam momenta far above the $\eta$ threshold. This may be due to systematical uncertainty of the data; however, it might be also attributed to isospin symmetry breaking effects not related to the meson mixing. Such an effect can be due to the different electromagnetic interaction in the two compared outgoing channels. This, however, is expected to be rather small due to large relative energy of outgoing particles. Another effect may originate from the differences in the wave functions of $^3\text{H}$ and $^3\text{He}$ nuclei. It may be as large as 10% as estimated within the simple model [19]. It may be, however, expected that such effects in the narrow beam momentum range will lead only to a constant shift of the cross sections.
ratio. Also a strong final state interaction between the intermediate $\eta$ and the $^3\text{He}$ nucleus cannot be excluded.

The simple model of Ref. [7] does not contain all the effects that may lead to a deviation of the cross sections ratio from the isospin symmetry predicted value of 2. On the other hand, the experimental ratio of the cross sections contains a systematical uncertainty of the relative acceptance determination. When comparing the model predictions to the experimental results these two factors can be combined together and treated as an overall normalization factor $N$, which multiplies the predictions. This leads to two free parameters, $N$ and $\theta_m$, necessary in the comparison of the model predictions with the data. As shown in Fig. 2(a), the $\chi^2$ value per degree of freedom ($\chi^2/n$ with $n = 3$) as a function of the two parameters, the normalization factor and the mixing angle, reveals a clear minimum. This global minimum is found by fixing the value of $N$ and fitting the $\theta_m$ value, which minimizes the $\chi^2/n$ with $n = 4$. Such procedure is repeated for several values of the normalization factor $N$, resulting in the dependence of $\chi^2/n$ shown in Fig. 2(b). The minimum is reached for $N = 1.15$ and $\theta_m = 0.006 \pm 0.005$ rad with the $\chi^2/n = 1.13$, where the quoted error corresponds to one standard deviation. The present result is much smaller than that obtained from the analysis of $\pi^- d \rightarrow pp \eta$ and $\pi^- d \rightarrow nn \eta$ reactions [11]. It is rather closer to the lower bound predictions of QCD based models. In order to improve the determination of the mixing angle more data are required. Also a more advanced model, which should include more sophisticated effects, will be necessary to draw a definite conclusion on the origin of isospin symmetry breaking. Such a model, based on $K$-matrix formalism, has been created by Green and Wycech [20]. Predictions based on that model might soon shed light on the isospin nonconserving nuclear dynamics.